TESIS DOCTORAL

New Insights in Idiosyncratic Risk

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Si los agradecimientos de esta tesis están es español (que no en castellano) es porque supongo que ésta será la única parte de la tesis que mi familia lea. O al menos la que leerá sin aburrirse. Y aunque la familia tiene semejante importancia, quiero expresar mi primer y más grande agradecimiento a Rosa y a David sin la ayuda de quienes esta tesis no habría sido posible. Más que directores han sido compañeros de trabajo entusiastas cuando las ideas escasearon, exigentes cuando la situación lo ameritó y, afortunadamente, comprensivos cuando las fuerzas me abandonaron. Y aunque esto no es poco, lo que más les agradezco es haberse emocionado igual o más que yo cuando las cosas salieron bien.

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“Al final, lo que importa no son los años de vida,
sino la vida de los años”

Abraham Lincoln (1808 – 1865)

A Sara, Ana, María, Andrea, Juan J., Gabriela,
Silvana, Jorge M., Gabriel, Antonio,
Violeta y Eloísa,

a quienes espero mostrar con el ejemplo que
hay muchas formas de vivir la vida;
unas mejores, otras peores,
pero la mayoría de ellas buenas.

Abstract

This thesis comprises three essays on the idiosyncratic risk anomaly. The first essay argues that the anomaly is not pervasive over investor time horizon and that it is only observed for short term investors. The empirical results suggest that features changing with the investment horizon such as risk aversion are relevant to explain the anomaly. The second essay links the anomaly to managerial decisions related to investment. It shows that including controls for investment and profitability in the cross-section of stock returns is sufficient to account for the anomaly. The empirical results suggest that the idiosyncratic volatility anomaly reflects the negative impact that corporate investment has over expected returns and that is not captured correctly by the Fama and French (1993) model. Also, that that this effect arises in part from investor mispricing and in part from risk exposure. The third essay highlights the relevance of economic regimes on the anomaly and, shows that the flight to liquidity evidenced by Acharya *et al.,* (2012) might be the reason why the anomaly is not observed after recession periods.

The main contributions of this thesis can be summarized as follows:

- The contributions in the second chapter are threefold. On the one hand, it proposes the co-existence of heterogeneous market players as the source of the idiosyncratic volatility anomaly. On the other hand, the paper proposes a methodology resulting in the estimation of one particular idiosyncratic risk measure for different group of investors defined according to their investment time horizon. Finally, it highlights the necessity finance discipline has of considering more complex mathematical methodologies that offer more realistic approximations to the complexity of financial markets. The major limitation of the paper is that no time horizon shorter than 2 days can be addressed given the daily character of the data used in the analysis.
- The third chapter offers an innovative approach based on the idea that the anomaly should be linked to managerial decision making and not necessarily

to investors. The empirical results show that the anomaly is fully accounted for when both investment and profitability controls are considered in the cross-section of stock returns. The results cast doubt on the generalized idea that the anomaly is related investor mispricing. They suggest that the anomaly is also constituted by a component of risk. The major limitation of the analysis is its inability to disentangle how much influence each component has in the anomaly.

- The third essay proves that the idiosyncratic volatility anomaly is conditional to the state of the economy and is not observed after recessions. The study stresses that during recessions investors move away from high firm specific risk stocks to cover their liquidity needs. It also shows that the effect of this flight to liquidity is larger than the one of the idiosyncratic volatility. Its main limitation is that the feature treated is not general enough to explain the anomaly across all economic regimes.

Resumen

Esta tesis está compuesta por tres ensayos sobre la anomalía del riesgo idiosincrático. El primero muestra que la anomalía no se extiende a todos los horizontes temporales de inversión y que sólo se observa para los inversores de corto plazo. Los resultados empíricos sugieren que las características que cambian con el horizonte de inversión tales como la aversión al riesgo son relevantes a la hora de explicar la anomalía. El segundo ensayo vincula la anomalía a las decisiones empresariales relacionadas con la inversión. Muestra que la inclusión de controles de inversión y de rentabilidad en la sección transversal de los retornos de las acciones es suficiente para explicar la anomalía. Los resultados empíricos sugieren que la anomalía de la volatilidad idiosincrática refleja el impacto negativo que la inversión tiene sobre los retornos esperados y que no es capturado correctamente por el modelo de Fama y French (1993). También sugieren que este efecto surge en parte por una valoración equivocada por parte de los inversores y, en parte por exposición al riesgo. El tercer ensayo subraya la importancia que los regímenes económicos tienen en la anomalía y muestra que la fuga a la liquidez evidenciada por Acharya *et al.,* (2012) puede ser el motivo por el cual la anomalía no se observa después de los periodos de recesión.

Las contribuciones principales de la tesis pueden resumirse así:

- En el primer capítulo las contribuciones giran alrededor de tres ejes. Por una parte, el ensayo propone que la coexistencia de agentes heterogéneos en el mercado como fuente de la anomalía de la volatilidad idiosincrática. Por otro lado, el estudio propone una metodología que resulta en la estimación de una medida de riesgo idiosincrático diferente para cada grupo de inversores definido de acuerdo a su horizonte de inversión. Finalmente, el ensayo subraya que la disciplina de las finanzas tiene la necesidad latente de considerar metodologías matemáticas más desarrolladas que ofrezcan aproximaciones más realistas a la complejidad de los mercados financieros. El límite mayor del ensayo es que, dado el carácter diario de los datos

utilizados en el análisis, no se puede considerar ningún horizonte temporal menor a 2 días.

- El tercer capítulo ofrece una aproximación innovadora cuya base es la idea de que la anomalía debería estar relacionada a las decisiones tomadas por la gerencia de la empresa. Los resultados empíricos muestran que la anomalía desaparece totalmente al incluir controles de inversión y de rentabilidad de manera conjunta en la sección transversal de los retornos de las acciones. Los resultados ponen en duda la idea generalizada según la cual la anomalía está relacionada en su totalidad a la valoración equivocada que los inversores hacen de algunas acciones y sugieren que la anomalía también tiene un componente de riesgo. La limitación mayor del análisis es su incapacidad para distinguir qué tanta influencia tiene cada uno de estos componentes en de la anomalía.
- El tercer ensayo demuestra que la anomalía de la volatilidad idiosincrática está condicionada por el estado general de la economía y que no se observa después de periodos de recesión. El estudio enfatiza el hecho de que durante las recesiones los inversores liquidan sus posiciones en acciones con mayor riesgo idiosincrático para cubrir sus necesidades de liquidez. También muestra que el efecto de esta fuga a la liquidez es mayor que el de la volatilidad idiosincrática. La mayor limitación del estudio es que la característica tratada en él no es lo suficientemente general para explicar la anomalía en todos los regímenes económicos.

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Chapter I: Introduction

Idiosyncratic risk has become a relevant topic in asset pricing because its information content is larger than theoretically anticipated. A main and controversial discussion in this framework is the idiosyncratic volatility anomaly or, the fact that portfolios with the lowest levels of idiosyncratic risk perform better than portfolios with the highest levels of it (Ang *et al.,* 2006 and 2009). The anomaly can be observed using portfolio sorting or Fama and MacBeth cross-sectional regressions and, is pervasive over sample periods in the American market. It is also observed across countries; firm specific risk and subsequent returns correlate negatively and significantly in 45 markets around the world among which 22 are emerging ones (Han *et al.,* 2011).

Even under relaxed assumptions, idiosyncratic risk should be irrelevant if investors have access to fully diversified portfolios. Furthermore, under-diversification models such as Merton (1987) show that an inability to diversify would theoretically imply a positive relationship between idiosyncratic risk and expected returns given that, aware of their additional exposure to risk, investors would require higher returns. Therefore, this simple observation has broad implications for asset pricing that represent new challenges for the field. In particular, it suggests that factorial asset pricing models such as the CAPM or the Fama and French (1993) model, commonly used to estimate the firm specific risk, might not be sufficient to explain stock returns.

With the exception of a paper by Eiling (2013) that argues the anomaly arises due to the model lacking pricing factors related to human capital, literature has systematically leave aside the discussion over the accuracy of factorial asset pricing models and its effects on the idiosyncratic risk issue. In contrast, it has provided a fruitful selection of plausible explanations based on known phenomena in the stock market that make appear the dismissal of factorial models as an excessive solution to the problem. Among these efforts, papers such as Kapadia (2006) and Boyer *et al*., (2010) explore the effect of investor preferences on the anomaly and show investors might accept lower returns for high firm-specific volatility stocks whose returns distributions offer desirable features like positive skewness or lottery-like payoffs (Bali *et al*., 2011). Microstructure issues such as returns reversals (Huang *et al.*, 2010) or trading non-synchronicity (Han and Lesmond, 2011) have also been linked to the anomaly but have been refuted by Chen *et al.,* (2012a) showing that the debate in this matter is sound and still developing. Gao *et al.,* (2012) argue the relationship between idiosyncratic volatility and expected returns depends on investor sentiment so that the anomaly is only observed during times following high investor sentiment periods. Finally, a large stream of literature argues that the anomaly is not arbitraged away because high idiosyncratic risk stocks are difficult to short (Boehme *et al*., 2009; Au *et al.*, 2009; Cao, 2009; Duan *et al.*, 2010).

This thesis intends to contribute to the growing research area that idiosyncratic risk anomaly brings to the asset pricing discipline. With the goal of increasing the knowledge on the firm specific risk and its relationship with subsequent returns, the thesis offers three studies approaching the anomaly from different perspectives. Given the puzzling nature of the anomaly addressed in this thesis the approaches are deliberately independent from one study to the other so that they do not necessarily follow a common thread in terms of underlying assumptions, of underlying paradigms or even of empirical methodologies. For instance, some readers might think the second chapter of this thesis suggests a research agenda based on behavioral approaches to the financial markets. They might then be surprised by the rational paradigm implied in the third and fourth chapters and conclude the thesis is somehow inconsistent. However, to give the broadest picture to an unexplained, recent and, puzzling observation, the subject should be approached from a variety of perspectives. In other words, no approach should be dismissed *ex-ante*. Overall, the thesis provides three self-contained papers that can be read jointly or separately, each highlighting different features of their common thread, the idiosyncratic volatility anomaly. The rest of this introduction intends to provide a general context for each of the chapters forming this thesis, to highlight the relevance of their contribution and, the main results driving it.

In the second chapter, it is argued that the co-existence of investors with different investment horizons might be linked the anomaly and that it should not be observed for long-term investors who are relatively more concerned about risk and tend to be more risk averse.

It was early shown by Levhari and Levy (1977) and Hawawini (1983) among others that systematic risk depends largely on the interval over which returns are measured. The use of an investment horizon different from the true one affects the estimation of idiosyncratic risk since, if the coefficients of the asset pricing models are biased, the residuals capturing the idiosyncratic component of risk should also be biased. In this framework, if all investors are considered homogeneous, the question becomes over which time horizon should returns be observed to fit the one investors truly have. If investors with different time horizons are assumed to co-exist in the market, the estimation of idiosyncratic risk should be different for each group of investors and, the anomaly should be studied separately for each of them.

In addition, the assumption that different groups of investors co-exist in the market leads to the question of how information is conveyed within the market. Literature on heterogeneous market models argues that each type of investor values information differently given their differing characteristics so that information spreads unevenly within the market. In turn, the uneven spread of information should introduce non-linearities into the return distributions of stocks. Given that these non-linearities are consistent with stylized facts like volatility clusters and fat tails authors such as Dacorogna *et al.,* (2001) and Los (2003) argue heterogeneity of market players is supported by financial data. More importantly, these authors conclude linear models are unfitted to analyze the risk – return relationship underlying the anomaly. To tackle this issue, the returns series of each stock and each pricing factor are decomposed using a wavelet multi-resolution analysis (WMRA). This methodology is able to assess non-linearities in the stock returns time series and is consistent with the hypothesis of co-existing investors with different time horizons.

The results obtained support the hypothesis of heterogeneous investors. In this sense, the initial non-linear relationship between idiosyncratic risk and expected returns is decomposed into two linear ones, each one related to a particular type of investors. Moreover, the results are consistent with the idea that investors assessment of risk changes from one group of investors to the other since the idiosyncratic volatility anomaly is only observed for short-term investors. The fact that the anomaly is not pervasive over time-horizons suggests that features changing with the investment horizon such as risk aversion are relevant in explaining the anomaly. Finally, the fact that the anomaly does not disappear for short-term investors shows that these investors are more likely driven by speculative motivations and intend to profit from short lived investment opportunity windows for which models such as the CAPM or the Fama and French (1993) model are not constructed.

The focus on the third chapter of this thesis shifts from investors to corporations and argues that the anomaly is fully accounted for when, following a basic argument advanced by valuation theory, controls for investment and profitability are included jointly in the cross-section of stock returns. The underlying argument is that the negative link between investment and expected returns (that accruals literature attributes exclusively to mispricing) has also a rational component attached to risk. In this sense, its main contribution is to offer a hypothesis neutral to investor expectations having two empirical testable implications and, that challenge the generalized idea that the anomaly might be related to investors' irrationality. In particular, it rules out the argument that the anomaly arises from mispricing driven by investor tendency to overreact to past accounting information (Jiang *et al.,* 2009) or by the overwhelming influence of irrational agents during times of high investor sentiment (Gao *et al.,* 2012).

By definition, managerial decisions should have a direct impact on the idiosyncratic component of risk for any firm. Therefore, the lack of studies on the relevance they might have on the anomaly is a good research opportunity. Conditional on managerial entrenchment, managers have the power to influence firm characteristics that have prediction power over expected returns. Among those, investment related characteristics including accruals, abnormal investment or asset growth seem potentially interesting since they correlate negatively with stock returns (Sloan, 1996, Cooper *et al.,* 2008). It is therefore plausible that the predictive power of idiosyncratic risk is actually driven by investment related firm characteristics whose effects are not totally captured by the factorial asset pricing models used to estimate the idiosyncratic volatility. If so, would this make the idiosyncratic volatility anomaly an issue of investor mispricing or rather an issue of risk? Fama and French (2006 and 2008) show that *a priori* there is no clear answer to this question. The negative relationship between investment and expected returns (that the literature on investment related anomalies attributes solely to mispricing) also arises under valuation theory that is neutral to investor expectations. However, some features about the idiosyncratic volatility anomaly provide two empirically testable hypotheses that would only support one of the views. On the one hand, in the mispricing approach controls for investment should be enough to account for the firm specific risk anomaly. In contrast, in the valuation theory approach it would be necessary to include additional controls for profitability. On the other hand, if valuation theory is supported, the joint controls for investment and profitability should hold also during high investor sentiment periods when irrational expectations tend to be more influential in the market.

Throughout the paper, a series of Fama MacBeth cross-setional regressions are estimated to test the effect that investment and profitability have on the idiosyncratic volatility anomaly. The results strongly support the hypothesis that valuation theory is most likely related to the anomaly since controlling only for investment is insufficient to account for the anomaly. However, it becomes non-significant when controls for profitability are included. Moreover, joint controls are equally effective during times of higher irrationality in several cases. Overall, the results suggest that the idiosyncratic volatility anomaly reflects the negative impact that corporate investment has over expected returns and that is not captured correctly by the factorial asset pricing models used to estimate the idiosyncratic risk. They also imply that it is likely that this effect arises in part from investor mispricing and in part from risk exposure.

Finally, the fourth chapter of this thesis points out the relevance of economic regimes on the anomaly and, shows that the flight to liquidity evidenced by Acharya *et al.,* (2012) might be the reason why the anomaly is not observed after recession periods.

As any other anomaly, the idiosyncratic volatility one has been forced to navigate its way to credibility through numerous tests. Surely anticipating this fact, Ang *et al.,* (2006 and 2009) provided several robustness tests for their surprising findings. Among these, authors included controls for business cycle showing that the anomaly is observed both during recessions and normal times. However, a recent paper by Gao *et al.*, (2012) shows it is only observed after periods of high investor sentiment casting doubt on the pervasiveness of the anomaly.[1] In this paper the anomaly is shown to be not pervasive over time disappearing after financial distress times. This fact seems consistent with the mispricing argument because recessions should be characterized by low investor sentiment so that arbitrageurs are expected to overrule sentiment investors and drive prices to reflect fundamental values so that no mispricing should take place. However, consistently with the third chapter of this thesis, sentiment should not be sufficient to account for the anomaly and an alternative explanation for the conditional character of the anomaly has to be provided.

The recently documented flight to liquidity phenomenon might offer a possible way to approach this conditionality. Indeed, Acharya *et al.,* (2012) enounce that during recessions financial agents have trouble dealing with liquidity shocks while they can usually address them during normal times. In response, investors have binding incentives to move from less to more liquid assets only during distress times. On the other hand, liquidity and idiosyncratic volatility are negatively correlated so that stocks with the highest levels of firm specific risk are more illiquid than stocks with the lowest levels of it (Spiegel and Wang, 2005). Then, during recessions investors would tend to shift from high to low idiosyncratic volatility stocks. This movement would tend to increase (decrease) the returns of stocks having low (high) firm specific risk. As liquidity shocks generating such a movement are absorbed by the market the correction in prices of these stocks goes against the idiosyncratic volatility anomaly, explaining why it is not observed after distress times.

The tool chosen to undertake this analysis is a multivariate Markov regime switching model meant to represent the asymmetric dynamic behavior of stocks over economic regimes. Being multivariate, the model adjusts a particular structure to the returns of the portfolio formed by stocks with the highest level of idiosyncratic risk and, another one to

[1] Note that although the third chapter of this thesis rules out the conclusion articulated by Gao *et al.*, (2012) that the anomaly is purely the reflection of investor mispricing, it does support the fact that the anomaly is not observed after periods of low investor sentiment.

the returns of the portfolio of stocks with the lowest level of it. Its switching nature implies each structure is also adjusted differently according to the regime. The results are highly satisfactory; during recessions the extreme idiosyncratic volatility quintiles portfolios are affected by liquidity shocks in opposite ways. Moreover, at distress times liquidity shocks are positively related to returns for stocks with low idiosyncratic risk while they do not have a significant effect for high firm specific risk stocks. Therefore, results support the hypothesis that the conditional effect of liquidity over stocks is also reflected into the idiosyncratic risk anomaly and provide a plausible explanation for the effect economic regimes have on it.

This introduction intended to provide a general overview of the idiosyncratic volatility anomaly as a prominent research field to which this thesis is related. The remainder of the thesis is organized as follows. Chapter II is constituted by the paper entitled “Time horizon trading and the idiosyncratic risk puzzle”. Chapter III corresponds to the paper entitled “Idiosyncratic Volatility Anomaly: Corporate Investment or Investors Mispricing?” Chapter IV to the one entitled “Idiosyncratic volatility, conditional liquidity and stock returns”. Chapter V highlights the contributions this thesis offers to the existent literature on the anomaly together with some elements for further research.

Chapter II: Time horizon trading and the idiosyncratic risk puzzle

Introduction

The relationship between idiosyncratic volatility and expected returns has become a major issue in recent research. [2] The current debate involves the evidence presented by Ang *et al.,* (2006), who found that by creating quintile portfolios sorted by stocks based on idiosyncratic risk levels, the portfolio with the highest level of idiosyncratic risk has significantly lower returns than the portfolio with the lowest level. This negative relationship is a controversial idea because it challenges both modern portfolio theory and under-diversification models (Merton, 1987). The former assumes essentially no link at all, and the latter assumes a positive link driven by a lack of investors' diversification capacity.

Possibly because of its controversial nature, the literature has been reactionary to the idiosyncratic volatility-expected returns puzzle, or the concept of the pricing ability of idiosyncratic risk. Thus, after the publication of the initial paper, most of the literature focused on showing that the puzzle was somehow incorrect and discussed the robustness of the results of Ang *et al*. (2006). For example, Bali and Cakici (2008) argued that the negative relationship either disappears or is not significant depending on the data frequency, the weighting scheme used to calculate average portfolio returns, the breakpoints used to sort portfolios' quintiles and the inclusion of small, illiquid stocks in the sample. A similar result was obtained by Fu (2009), who argued that Ang *et al.* (2006) mistakenly concluded that the link between idiosyncratic risk and expected returns is negative by assuming that idiosyncratic volatility is persistent. Using an EGARCH model

[2] Although the recent debate has renewed the discussion on the predictability of returns using idiosyncratic risk, the issue has been extensively discussed in the past. Both Douglas (1969) and Lintner (1965) find significant explanatory power of the variance of the residuals from a market model in the cross-section of average stock returns. Miller and Scholes (1972) and Fama and Macbeth (1973) argue for statistical problems. Finally, Lehmann (1990) reaffirms Douglas' results after a careful econometric revision.

to account for heteroscedasticity in idiosyncratic volatility, Fu finds a positive and significant result.

Following the initial criticisms, Ang *et al.* (2009) proved the robustness of their puzzle by examining not only American data but also data from all other G7 countries (Canada, France, Italy, Germany, Japan, US and UK). Again, they found the same negative and significant relationship between idiosyncratic volatility and expected returns for all countries. Furthermore, the American data results were shown to be robust to different weightings for the formation of portfolios and to the use of different periods to compute idiosyncratic risk. This second paper marked a turning point in the literature, and the puzzle gained credibility after its publication, turning the discussion to possible reasons for this relationship. Our paper is part of this literature and provides new insights into this puzzle.

One of the relatively recent hypotheses is that the so-called volatility returns puzzle is driven by investors' heterogeneity in the financial markets. In this sense, Brandt *et al.* (2010) argue that the puzzle is driven by retail investors, which have a special preference for stocks with high idiosyncratic volatility compared with institutional investors, who tend to minimize their exposure to this type of asset. However, some authors argue that mutual fund managers prefer stocks with high idiosyncratic risk (Falkenstein, 1996) and that when they are willing to increase their risk, they increase their portfolio's exposure to idiosyncratic risk (Huang *et al.,* 2011). In this paper, we assume the perspective of heterogeneity of market players, focusing specifically on heterogeneity in investors' time horizons. Financial markets comprise investors and traders with different investment time horizons: market makers, intraday traders, day traders, short-term traders and long-term traders. The aggregation of the activities of all traders is what ultimately generates prices. The heterogeneity assumption implies that the true dynamic relationship between the various aspects of market activity is only revealed when the market prices are decomposed by different time scales or investment horizons.

Both heterogeneity of investors and time horizons are important concepts for asset pricing. On the one hand, empirical stylized facts such as fat tails and volatility clusters are difficult to explain in the context of homogeneous investors, whereas they naturally arise in computational markets with different types of investors (Lévy *et al.,* 2000, Gil-Bazo *et al.*, 2007). On the other hand, empirical tests of risk loadings depend largely on the time interval, and systematic risk is biased when using a shorter-term investor's time horizon rather than the true risk (e.g., Levhari and Levy, 1977). By extension, considering heterogeneous investors based on differences in time horizons should affect idiosyncratic volatility estimation and could thus be expected to enhance the study of this puzzle. However, this approach entails the problematic issue of separating investor classes and their influence on idiosyncratic volatility and returns.

Wavelet multiresolution analysis (WMRA) is useful for differentiating time horizons. WMRA allows for the decomposition of a time series into different time horizons, called time scales, each of which correspond to a particular frequency. Because different investors have different trading frequencies, the first scale should yield information on short-term investors, whereas the higher scale should provide information on long-term investors (Müller *et al.,* 1997, Gençay *et al*., 2005 and 2010 and, In and Kim 2006). In this context, asset allocation depends on investors' time horizons (In *et al.,* 2011), asset pricing models yield different results for each time scale (Gençay *et al*., 2003 and 2005), and the negative risk-return link shown by Ang *et al.* (2006, 2009) may not be valid for all investors.[3]

Alternative explanations state that the puzzle is observed because stocks with the highest levels of idiosyncratic risk are difficult to short-sale; thus, pessimistic information does not flow to these stock prices (Boehme *et al.,* 2006, Asquith *et al.,* 2005). An additional proposal by Boyer *et al.* (2010) argues that stocks with higher idiosyncratic risk offer larger probabilities of an extreme positive return and thus have lower returns. Finally, in Berrada and Hugonnier (2012), the puzzle is related to firms' cash flow growth rate. Although these authors suggest that explanations for the puzzle are related to the specific

[3] Wavelet analysis is relatively new in economics and finance, although the literature on wavelets is growing rapidly. Applications in these fields include the study of systematic risk in the capital asset pricing model (Gençay *et al.* 2003 and Rhaiem *et al.*, 2007), the multi-scale relationship between stock returns and inflation (Kim and In, 2005), the relation between returns and systematic co-kurtosis and coskewness (Galagedera and Maharaja, 2008), a multiscale hedge ratio (In and Kim 2006), studies in portfolio management (In *et al.* 2008 and Bowden and Zhub, 2010) and portfolio allocation (Kim and In 2010), the analysis of co-movements in stock markets (Rua and Nunes, 2009), Value at Risk measures (Fernandez, 2005) and credit portfolio losses (Masdemont and Ortiz-Gracia, 2011).

characteristics of the stocks or firms with the highest level of idiosyncratic risk, we argue that investors' characteristics are also relevant.

The main goal of this paper is to separate short-term investors and long-term investors through a WMRA using daily data to study separately their influence in monthly stock prices. Therefore, we can analyze the puzzle for each group separately. To the best of our knowledge, this approach has not been applied previously to the idiosyncratic volatility-expected returns puzzle. We hypothesize that the negative relationship is driven by short-term investors who do not necessarily follow the typical mean-variance logic that makes Ang *et al.*'s (2006 and 2009) result puzzling. Our results confirm that the puzzle disappears as the wavelet scale increases; the idiosyncratic risk-returns relationship turns positive at larger scales, indicating that investors with long-term horizons should not worry about the puzzle compared with those with short-term horizons. Moreover, our approach provides an explanation for all stocks (not only the riskiest ones) and is robust to changes in wavelet family, idiosyncratic risk estimators and coskewness or liquidity factors.

The remainder of the paper is organized as follows. Section 2 provides a general discussion of empirical tests of asset pricing models that justifies the use of wavelet decomposition for this particular analysis. Preliminary evidence of the puzzle in our sample is provided in section 3. Section 4 describes wavelets and multiresolution analysis methodology and the empirical results. Section 6 analyzes the robustness of our findings, and Section 7 concludes the paper.

Empirical tests of asset pricing models, time horizons and wavelets

Given the continuous nature of price formation, determining the correct time interval to empirically test asset pricing models is impossible. This is why any empirical study is subject to critiques on data frequency such as the ones made about Ang *et al.* (2006 and 2009) by Bali and Cakici (2008), who show that using monthly instead of daily data achieves different conclusions about the idiosyncratic risk-expected returns relationship. Nevertheless, it is well known that systematic risk depends largely on the interval over which returns are measured (Levhari and Levy, 1977, Hawawini, 1983, Handa *et al.,* 1993, Brailsford and Josev, 1997, Brailsford and Faff, 1997, among others) [4]. Furthermore, the so-called Epps effect (Epps, 1979) shows that stock return correlations decrease as the return interval increases. As risk measures change with the return interval (i.e., implied investor time horizon), the idiosyncratic risk estimation is expected to change, introducing the need to study the puzzle for different time horizons.

From a statistical point of view, a possible reason for the mixed evidence on empirical tests of asset pricing models might be the divergence between theoretical assumptions used in the construction of models and the empirical evidence itself. In an efficient market, asset prices reflect all relevant and available information, and any news affecting them is simultaneously and immediately incorporated into prices. New information has to be independent and random to avoid being anticipated and immediately translated to prices. Thus, the instantaneous adjustment implies the independence of price increments and a singular time horizon. However, stylized facts, such as volatility clustering and fat tails, contradict this *i.i.d.* assumption. In this context, heterogeneous agent models state that the market is formed by investors with different characteristics who judge which information is relevant according to their nature.[5] Specifically, we consider market participants with different time horizons. From this perspective, information is diffused unevenly, the independence of price increments does not hold, and asset prices reflect a combination of long- and short-term valuation processes. [6] Thus, financial risk depends not only on time but also on the particular investment horizon; financial risk is both time varying and time-scaling (Los, 2003).

[4] In particular, the beta of thinly traded securities increases as the return interval rises, whereas the beta of frequently traded securities falls. Furthermore, the estimated beta of highly capitalized firms decreases as the return interval increases, whereas the beta of low-cap firms increases.

[5] Heterogeneous agent models (Müller *et al.,* 1993 and 1997, LeBaron, 2000) are theoretical explanations for empirical stylized facts based on the existence of differences in investors. Müller argues that differences can be observed in perceptions, institutional constraints, risk profiles, prior beliefs and geographical location. We focus on the idea of differences in time horizons because it offers the possibility of the mathematical treatment we show.

[6] O'Hara (2003) discusses the impact of diverging information within the classical asset pricing model assumptions. Her main conclusion is that asymmetric information derives from a group of uninformed (noise) traders who, even if they systematically lose to better-informed ones, make portfolio choices so that their risk exposure to wins of informed investors is lower.

So far, we have introduced three concepts: time horizons, frequency and scale. As shown, time horizons determine frequencies. Scale and frequency are directly related so that time-scaling risk corresponds to the notion of risk being assessed differently by investors with various time horizons. Differing investment horizons violate the independence assumptions and thus introduce global (in opposition to serial) dependences on the return series. These are very difficult to assess with ARMA or GARCH family models because correlations are transient or have varying frequencies. Therefore, to analyze risk in this framework, the analytical tools used to allow different time scales in most of the financial literature must be changed. A recent major development regarding the time-scales issue in financial data is multiresolution analysis from wavelet decomposition (Mallat, 1989), which appeared in the late 1980s. The next section summarizes the main features of this statistical tool, which provides an additive decomposition in which, instead of differentiating a trend into a seasonal and a cyclic component, a time series is viewed as a sum of time scales accounting for local changes.

2.1.1. Wavelets methodology and multiresolution analysis

Considering global (long-term) dependence on market returns shows that returns series are non-stationary and that the assessment of financial risk involves more than the first two moments of the distribution (Mandelbrot, 1972). To obtain evidence for any form of time dependence, it is necessary to simultaneously have both distributional and the time-localized evidence.

The first technique to develop information on frequencies for a given time series was Fourier analysis. The Fourier transform is not suitable for financial data because it is only meaningful when the time series is stationary and does not have sudden changes. Furthermore, the transform loses all time-dependence information so that global dependences are impossible to isolate using this technique.

Wavelet transform is similar to the Fourier transform but does not lose time information because wavelets are localized by both time and frequency. In addition, wavelets are functions with finite time support so that they are able to cope with sudden changes in signals. We are interested in a particular feature of wavelet analysis: the wavelet multiresolution analysis. This technique divides the time-frequency space into frequency bands separated by multiples of 2^j, with time support divided by 2 as frequency increases. To perform WMRA, a time series S_0 is decomposed into a blurred approximation S_i (long-run horizon) and the remaining details D_i (short-run horizons), where S_i and D_i are orthogonal to each other. The resulting series in the time domain is the contribution of frequency i to the original series or the component of the original series that has frequency j (Norsworthy *et al*., 2000).

Mallat (1989) developed a method to perform the WMRA of a signal through simple linear filters that separate its high frequency elements (corresponding to the details) from its low frequency ones (corresponding to the blurred approximation). Frequency separation is performed using two linear filters, each blocking a particular frequency (high or low) while letting the other (low or high) pass through. The series are recomposed into the time domain using the corresponding quadrature mirror filters. There are many wavelet transform classes and wavelet families with particular properties. We use the Maximum Overlap Discrete Wavelet Transform (MODWT) and the Haar wavelet family (Haar, 1910), for which the filter coefficients are $H_0 = \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ for the low pass filter and $G_0 = \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ for the high pass filter. Both the scale series and the details series are obtained using the quadrature mirror filters $\overline{H_0} = \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ and $\overline{G_0} = \left[\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right]$. Gençay *et al.* (2002) provide a comprehensive analysis of wavelets and several similar filter-related methodologies applied to economics and finance.

Data description and preliminary evidence

In this section, we follow the procedure of Ang *et al.* (2006) to confirm the idiosyncratic volatility-expected returns puzzle for our sample before addressing any time-horizon issues using wavelets. Our database includes daily returns of all stocks in the CRSP

(*Chicago Research Stock Prices*) with more than 17 observations in a month for the NYSE, AMEX and NASDAQ markets from July 1963 to December 2009. For each month, we sort stocks according to their idiosyncratic volatility, defined as the standard deviation of the residuals ($\sigma_{\varepsilon_t^i}$), in the three-factor model of Fama and French (1993):

$$r_t^i = \alpha^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \varepsilon_t^i \,, \qquad [1]$$

where r_t^i is the stock return in excess of the risk-free rate, and $\{MKT_t, SMB_t, HML_t\}$ represent the market, size and book-to-market factors.[7] Once we have sorted the stocks into quintiles, with the first containing stocks with the lowest risk and the last containing the highest risk, we form portfolios and hold them for one month. The corresponding portfolios are value weighted and rebalanced month by month.[8]

Table 1 reports the results obtained by replicating the process of Ang *et al.* (2006) using data from July 1963 to December 2009. In columns, we present the average returns, standard deviation, and alphas for portfolios sorted based on idiosyncratic volatility. All of these are reported in monthly percentages. Alphas CAPM correspond to Jensen's alphas calculated with respect to the CAPM and Alphas FF with respect to the three-factor model. The t-statistics are reported in brackets. The row [5-1] is the difference between portfolio 5 and portfolio 1, with the Newey-West t-statistic also reported in brackets.

The main patterns reported by Ang *et al.* (2006) appear in our sample. Average returns of portfolios sorted by idiosyncratic volatility display an inverse U-shaped form that increases in the middle quintiles; returns rise from 0.88% in quintile 1 to 1.10% in quintile 3 and then drop to 0.22% in quintile 5. The difference [5-1] is, on average, -0.66% per month. It is negatively significant at 10% when using the Newey-West t-statistic and at 5% when using White's Heteroskedasticity-Consistent t-statistic. Moreover, Jensen's alphas are

[7] They have been obtained from Kenneth French's website http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library html.
[8] We also verify the puzzle using 6 months for the regression in equation [5] to address the critique of error-in-variance exposed by Malkiel and Xu (2002). The results do not change. They are available upon request.

Table 1: Returns of portfolios sorted by idiosyncratic risk

This table reports the results we obtain by forming portfolios of quintiles according to idiosyncratic risk using data from July 1963 to December 2009. Quintile 1 corresponds to the lowest idiosyncratic risk portfolio, and quintile 5 corresponds to the highest idiosyncratic risk. Returns and standard deviation (Std Dev) are reported as monthly percentages. The row [5-1] is the difference between portfolio 5 and portfolio 1. Alphas CAPM correspond to Jensen's alphas calculated with CAPM, and Alphas FF correspond to the alphas calculated with the three-factor model. Newey-West t-statistics are reported in brackets, and p-values are reported in parentheses. * denotes significance at the 10% level, and ** denotes significance at the 5% level.

Quintile	Returns	Std Dev	Alphas CAPM	Alphas FF
1	0.88	3.58	0.12 [1.63] (0.10)	0.04 [0.67] (0.50)
2	1.01	4.55	0.13 [2.45] (0.01)	0.09 [1.93] (0.05)
3	1.10	5.83	0.11 [1.30] (0.19)	0.13 [1.85] (0.07)
4	0.88	7.56	-0.23 [-1.54] (0.13)	-0.22 [-2.08] (0.04)
5	0.22	9.19	-0.96 [-4.01] (0.00)	-1.00 [-5.69] (0.00)
[5-1]	-0.66* [-1.82] (-0.07)		-1.08** [-3.66] (0.00)	-1.04** [-4.97] (0.00)

positive for the initial three portfolios and become negative starting with the fourth. Both [5-1] differences in Alphas CAPM and in Alphas FF are negative (-1.08% and -1.04%, respectively), showing that the puzzle appears even after controlling for risk. The presence of similar patterns in our results provides evidence of the robustness of the results of Ang *et al.* (2006 and 2009). Their main conclusions hold for a longer period and are not modified by the particularly unstable time characteristic of the recent years in our sample.

Because asset pricing models impact the estimation of idiosyncratic volatility, another question is whether the puzzle is robust to different models. We replace the Fama and French three-factor model with the Carhart (1997) model so that the idiosyncratic volatility used to sort the stocks is the standard deviation of the residuals from the following equation:

$$r_t^i = \alpha^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \beta_{MOM}^i MOM_t + \varepsilon_t^i, \qquad [2]$$

where r_t^i is the stock excess returns, and $\{MKT_t, SMB_t, HML_t, MOM_t\}$ represents the market , size, book-to-market and momentum factors.

Table 2: Evidence of the puzzle using Carhart (1997)

This table reports the results using the Carhart (1997) model for the data sample from July 1963 to December 2000. Quintile 1 corresponds to the lowest idiosyncratic risk portfolio, and quintile 5 corresponds to the highest idiosyncratic risk. Returns and standard deviation (Std Dev) are reported as percentages. [5-1] is the difference between portfolio 5 and portfolio 1, and [5-2] is the difference between portfolio 5 and portfolio 2. Alphas CAPM correspond to Jensen's alphas calculated with CAPM, and Alphas FF correspond to the alphas calculated with the Carhart (1997) model. Newey-West t-statistics are reported in brackets, and p-values are reported in parentheses. * denotes significance at the 10% level, and ** denotes significance at the 5% level.

Quintile	Returns	Std Dev	Alphas CAPM	Alphas FF4
1	0.88	3.58	0.11 [1.64] (0.10)	0.05 [0.78] (0.43)
2	1.02	4.55	0.14 [2.62] (0.00)	0.10 [2.27] (0.02)
3	1.11	5.82	0.11 [1.32] (0.19)	0.12* [1.73] (0.08)
4	0.92	7.56	-0.20 [-1.30] (0.20)	-0.18 [-1.53] (0.13)
5	0.27	9.28	-0.92 [-3.88] (0.00)	-0.98 [-5.52] (0.00)
[5-1]	-0.61* [-1.70] (0.09)		-1.03** [-3.55] (0.00)	-1.02** [-4.73] (0.00)
[5-2]	-0.75** [-2.47] (0.01)		-1.06** [-4.05] (0.00)	-1.08** [-5.70] (0.00)

Table 2 illustrates the results from Carhart's model. As in the previous table, returns and standard deviations are monthly averages in percentages, and alphas for CAPM and Carhart's model are tabulated. All t-statistics are in brackets.

Qualitatively, the results for alphas, returns and standard deviations are similar, and the [5-1] returns difference is equal to -0.61% but is only significant at the 10% level. However, the puzzle can be verified using the second quintile portfolio because the fact that a riskier portfolio yields lower returns also appears for the fifth and second quintiles. The negative link is significant at the 5% level if one considers the [5-2] returns difference.[9] Therefore, controlling for momentum is not relevant to explaining the puzzle.[10] At this point, we have proven the puzzle is present in our sample and that it is robust to changes in idiosyncratic risk estimation. We can now turn to the implications of time horizons on idiosyncratic risk-expected returns relationship.

Methodology and empirical results

In this section, we briefly describe the process followed for the wavelet approach to shed light on the idiosyncratic risk-expected return puzzle. We distinguish several time scales, each corresponding to a group of investors with a particular and homogeneous time horizon. Our hypothesis is that investors value information according to their investment horizon; therefore, for each group, a different idiosyncratic risk-expected return link may be observed.

Theoretically, once the efficient market hypothesis is dropped by introducing differing time horizons, the number of investor groups accounted for is unlimited. This number is also not limited by the WMRA nature. Therefore, the first stage of the analysis is to determine the number of time scales considered. Many articles on portfolio rebalancing limit the number of time scales to the maximum possible number before the rebalancing (Gençay *et al*., 2003 and 2005). With daily data, a one-level MRA divides the data into two investment horizons (2 to 4 days and more than 4), a two-level MRA divides the data into three investment horizons (2 to 4 days, 4 to 8 days and more than 8), and a three-level

[9] Although not reported here, the significance of the [5-1] difference is sensitive to the time period considered. Only for three months in the sample from November 1990 to December 2009 is the difference not significant at either the 5% or 10% levels. These results are available upon request.
[10] The same conclusion was found by Ang et al. (2006) using another methodology. They included a double-sorting control for momentum, after which the puzzle still holds.

MRA divides the data into four investment horizons (2 to 4 days, 4 to 8 days, 8 to 16 days and more than 16).[11] Therefore, the maximum number of time scales is three because our rebalancing is performed monthly (i.e., approximately 17 to 20 days).

For each MRA level and time scale, we calculate Fama and French's 3-factor model and sort the stocks according to the standard deviation of the residuals. Then, we compute the five portfolio quintiles and focus our attention on the difference in the returns from the fifth to the first quintile. This procedure produces two time scales (D1 and S1) for the one-level MRA, three time scales (D1, D2 and S2) for the two-level MRA and four time scales (D1, D2, D3 and S3) for the three-level MRA. Because WMRA is performed recursively, both the D1 and D2 time scales are the same for an MRA of any level.

The finer scale of the MRA can be expected to isolate the behavior of short-term investors (e.g., technical analysts).[12] By extension, fundamentalists' behavior should be reflected on a coarser scale. Because the idiosyncratic risk-expected returns puzzle cannot be explained by classical portfolio theory, the empirical characteristics of financial data provide evidence supporting the hypothesis of multiple types of investors in the market, and long-term investors are expected to follow fundamentals (i.e., they are close to the representative agent in classical asset pricing models). This is consistent with the findings of Gençay *et al.* (2005), which show that the relationship between the return of a portfolio and the systematic risk measure becomes stronger as the investment horizon increases. Thus, we expect the negative idiosyncratic volatility-expected returns liaisons to disappear at a coarser scale.

We begin with the simplest case, in which there are only two groups of investment horizons, short-term (2 to 4 days) and long-term (more than 4 days), corresponding to D1 and S1, respectively. The decomposition linked to this hypothesis is the one-level MRA, for which results are reported in Table 3. The structure of this table is analogous to the

[11] Notice that, as stated previously, the time scale increases by multiples of 2^j, and we are working with daily data. Therefore, the time scale increases by 2^j days each time: 2-4 days, 4-8 days, 8-16 days and so on.

[12] It has been shown that technical analysis is mostly used for short-term forecasting (Frankel and Froot, 1990). However, we prefer to let the exact nature of investors be an open issue and limit our classification to the relative frequency of trading of each group of investors considered in the MRA. This is because we assume that investors use all tools available for decision making no matter how frequently they trade.

initial tables but duplicates columns to display information for both investor groups. The first four columns correspond to the analysis for short-term investors (D1). Returns and standard deviations are average monthly percentages across portfolios, and the last two columns for each group display Alphas CAPM and FF. The last four columns illustrate the results for long-term investors. All t-statistic values are tabulated in brackets.

Table 3: One-level MRA results. Haar wavelet.

The tables display MRA results for two groups of investment horizons: short-term and long-term horizons, from July 1963 to December 2009. Quintile 1 corresponds to the lowest idiosyncratic risk portfolio, and quintile 5 corresponds to the highest idiosyncratic risk. Returns and standard deviation (σ) are reported as percentages. [5-1] is the difference between portfolio 5 and portfolio 1, and [5-2] is the difference between portfolio 5 and portfolio 2. Alphas CAPM correspond to Jensen's alphas calculated with CAPM, and Alphas FF correspond to the alphas calculated with the three-factor model. Newey-West t-statistics are reported in brackets, and p-values are reported in parentheses. * denotes significance at the 10% level, and ** denotes significance at the 5% level.

	Short-term horizon: 2 to 4 days				Long-term horizon: more than 4 days			
Quintiles	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**
1	-0.09	0.38	-0.06 [-11.11] (0.00)	-0.06 [-12.11] (0.00)	1.01	3.45	0.21 [2.93] (0.00)	0.11 [1.74] (0.08)
2	-0.15	0.51	-0.11 [-14.27] (0.00)	-0.11 [-14.64] (0.00)	1.24	4.26	0.32 [6.41] (0.00)	0.27 [5.88] (0.00)
3	-0.26	0.66	-0.23 [-15.31] (0.00)	-0.21 [-15.49] (0.00)	1.40	5.29	0.37 [5.73] (0.00)	0.39 [6.22] (0.00)
4	-0.47	0.86	-0.42 [-13.47] (0.00)	-0.41 [-13.55] (0.00)	1.70	6.97	0.52 [3,65] (0.00)	0.63 [5.61] (0.00)
5	-1.04	1.23	-0.99 [-13.08] (0.00)	-0.97 [-13.20] (0.00)	1.91	8.84	0.61 [2.83] (0.00)	0.72 [4.08] (0.00)
5-1	-0.96** [-12.34] (0.00)		-0.93** [-12.50] (0.00)	-0.90** [-12.61] (0.00)	0.90** [2.64] (0.01)		0.40 [1.51] (0.13)	0.61** [2.90] (0.00)
5-2	-0.90** [-12.23] (0.00)		-0.88** [-12.33] (0.00)	-0.85** [-12.42] (0.00)	0.67** [2.31] (0.02)		0.40 [1.51] (0.13)	0.45** [2.48] (0.01)

The results support our hypothesis in that the puzzle is only present for short-term investors. For this group (D1), the [5-1] returns difference is significantly negative and equal to -0.96%. Furthermore, the puzzle remains after controlling for CAPM and FF risk factors; [5-1] differences in both models' alphas are negative (-0.006 in both cases). For long-term investors (S1), the puzzle disappears, and the [5-1] difference takes a positive and significant value of 0.90%.

A noteworthy result is that average monthly returns sorted by idiosyncratic risk do not exhibit an inverse U-shaped form. For short-term horizons, returns decrease linearly from -0.09% in the first quintile to -1.04% in the last. For long-term horizons, returns increase from 1.01% for the portfolio with the lowest idiosyncratic risk to 1.91% for the riskiest one. These patterns support the idea of an asset price formation resulting from two heterogeneous groups. In this sense, the inverse U-shaped return pattern indicates a nonlinear relationship between idiosyncratic risk and expected returns. We argue that this relationship is caused by the interaction of investors with dissimilar time horizons. Alternatively, returns' U-shaped form could be the result of a missing risk factor. However, we think our results support our working hypothesis, because a non-linear liaison related to a missing risk factor should be reflected for both short- and long-term investors. In addition, our results are corroborated by a recent study by Cao and Xu (2010), who decomposed the idiosyncratic volatility into long-run and short-run components and found the existence of a negative short-run effect.

Another significant but challenging fact is that for short-term horizons, all portfolios have negative returns (-0.09%, -0.15%, -0.26%, -0.47% and -1.04%). We hypothesize that the highest frequency scale involves isolating short-term strategies, which have different objectives from the typical mean-variance strategy, and that the negative results are random. Alternative explanations can be provided by O'Hara's idea of a group of (uninformed) investors persistently losing to the other group (informed) while rationally minimizing their risk exposure (O'Hara, 2003) or by the Fractal Hypothesis Market, in which the negative signs can be understood as evidence of the greater likeliness of crowd behavior in short-term movements.[13]

[13] Adopting O'Hara's idea would require separating short-term investors from uninformed ones. However, we do not have sufficient evidence to do this. The Fractal Market Hypothesis proposes that information is more closely related to market sentiment and technical factors in the short term than in the long term and that short-term price movements are likely to be the result of crowd behavior (Blackledge, 2010).

In contrast, before the decomposition into time horizons, the negative link between expected returns and idiosyncratic risk is driven by a significant drop of -75% (that is, 0.22 - 0.88) in returns from the fourth to the fifth quintile. In this sense, Brandt *et al.* (2010) and Han and Kumar (2009) report that the negative idiosyncratic volatility-expected returns relationship is stronger for the small, low-priced stocks typically held by retail investors.[14] However, in our analysis, retail investors may be in either of the two categories of investors, but they are most likely represented in both groups. For short-term investors, the notorious drop in the last quintile portfolio return remains. Other explanations relate this notorious drop to the type of stocks classified in the fifth quintile (e.g., Asquith *et al.,* 2005, Boehme *et al.,* 2006). Because changes from quintile to quintile are similar for short-term investors (increases of approximately 100%), showing the linear relationship described previously, the special features of stocks in the fifth quintile that cause the puzzle are not supported. These stocks are represented both in the short-term series (D_i) and in the long-term series (S_i). If they drove the puzzle, their effects should be observed in both groups. Although these stocks may be related to the puzzle because of some special feature, only the movements of short-term investors explain the appearance of a negative relationship between idiosyncratic risk and returns.

Robustness

In this section, we study whether our main result (that the puzzle is present in the short horizon but not in long horizons) is robust to several estimators of idiosyncratic risk and different definitions of short-term investors. The wavelet family may influence the results because it dictates the length of the MRA filter. A longer filter implies a larger adaptability to complex time series in the WMRA. In Table 4, we display the one-level MRA for two

[14] The authors argue that retail investors are especially interested in these stocks because of their speculative character: high skewness and high volatility.

groups of investors, short-term and long- term investors, corresponding to D1 and S1, respectively, for a Daubechies 8 wavelet.[15]

Table 4: One-level MRA results. Daubechies 8 wavelet.

The tables display MRA results for two groups of investment horizons: short-term and long-term horizons, from July 1963 to December 2009. Quintile 1 corresponds to the lowest idiosyncratic risk portfolio, and quintile 5 corresponds to the highest idiosyncratic risk. Returns and standard deviation (σ) are reported as percentages. [5-1] is the difference between portfolio 5 and portfolio 1, and [5-2] is the difference between portfolio 5 and portfolio 2. Alphas CAPM correspond to Jensen's alphas calculated with CAPM, and Alphas FF correspond to the alphas calculated with the three-factor model. Newey-West t-statistics are reported in brackets, and p-values are reported in parentheses. * denotes significance at the 10% level, and ** denotes significance at the 5% level.

	Short-term horizon: 2 to 4 days				Long-term horizon: more than 4 days			
Quintiles	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**
1	-0.11	0.38	-0.08 [-11.30] (0.00)	-0.08 [-12.29] (0.00)	1.00	3.46	0.20 [2.79] (0.00)	0.10 [1.58] (0.11)
2	-0.17	0.53	-0.13 [-13.48] (0.00)	-0.13 [-13.96] (0.00)	1.25	4.27	0.34 [6.27] (0.00)	0.28 [5.73] (0.00)
3	-0.29	0.66	-0.25 [-15.62] (0.00)	-0.24 [-15.84] (0.00)	1.32	5.22	0.30 [4.82] (0.00)	0.34 [5.39] (0.00)
4	-0.54	0.92	-0.48 [-13.19] (0.00)	-0.46 [-13.40] (0.00)	1.63	6.98	0.46 [3.53] (0.00)	0.56 [5.41] (0.00)
5	-1.18	1.13	-1.12 [-12.85] (0.00)	-1.09 [-12.88] (0.00)	1.75	8.96	0.45 [2.15] (0.03)	0.57 [3.27] (0.00)
5-1	-1.07** [-12.12] (0.00)		-1.04** [-12.19] (0.00)	-1.01** [-12.24] (0.00)	0.75** [2.24] (0.03)		0.26 [0.99] (0.32)	0.47** [2.25] (0.02)
5-2	-1.01** [-12.07] (0.00)		-0.98** [-12.08] (0.00)	-0.96** [-12.10] (0.00)	0.50* [1.74] (0.08)		0.12 [0.51] (0.61)	0.29 [1.59] (0.11)

The figures reported in Table 4 show a [5-1] returns difference that is significant and negative for short-term investors (-1.07%) but significant and positive (0.75%) for long-term investors. Furthermore, for both groups, idiosyncratic risk and returns are linearly

[15] Although many other possibilities exist, in this paper, we consider only the Haar and the Daubechies wavelet families. We consider the Haar family our benchmark because many of the previous studies available on risk loadings in asset pricing models use it. Keeping the same family facilitates comparisons. The Daubechies family is a natural extension in that the Haar wavelet is the Daubechies wavelet of minimum length. It is also a common wavelet family for studies in economics and finance. See, for example, Fan and Gençay (2010) or Huang and Wu (2008).

related; returns monotonically decrease from -0.11% to -1.18% for D1 and monotonically increase from 1.00% to 1.75% for S1. The consistency of the results across different wavelet families leads us to the conclusion that the wavelet family does not drive our conclusions.

In Tables 5 and 6, we present a two-level and a three-level MRA, respectively, to include more investor groups. As explained in section 4, D1 represents the 2- to 4-day investors, D2 represents the 4- to 8-day investors, D3 represents the 8- to 16-day investors, and S2 and S3 represent long-term investors. The tables follow the same structure as the previous ones but include additional columns with the information for D2, D3, S2 and S3 according to the MRA level considered.

For alternative definitions of short run, the puzzle remains; risks and returns are significantly and negatively related for both D2 (-0.30%) and D3 (-0.24%). In these cases, portfolios' returns are also negative for all short-term horizon definitions. However, both the magnitude of the negative relationship constituting the puzzle and its significance level diminish as the investment horizon increases. Furthermore, we build a monthly S1 series for stocks with no missing values over the sample period and determine the influence of short-term investors by comparing it to the original monthly series. We find that, even if short-term movements are relevant in the daily decomposition, compounding the daily long-term series (S1) produces basically the same series as the original. In fact, over the whole period, only 21% of the stocks have points outside the one-standard-deviation interval. Of these, 73% have only one point outside the interval, and 100% have 4 or fewer points. Monthly S1 and the original series are very similar in terms of the mean and standard deviation and have very large correlations (0.99 in mean). However, there are marked differences in terms of skewness and kurtosis for some of the stocks.[16] Thus it is possible that stocks in the highest quintile are stocks with a higher coskewness values so

[16] Results are available upon request.

Table 5: Two-level MRA results. Haar wavelet.

The tables display MRA results for three groups of investment horizons: 2- to 4-day horizons, 4- to 8-day horizons and long-term horizons, from July 1963 to December 2009. Quintile 1 corresponds to the lowest idiosyncratic risk portfolio, and quintile 5 corresponds to the highest idiosyncratic risk. Returns and standard deviation (σ) are reported as percentages. [5-1] is the difference between portfolio 5 and portfolio 1, and [5-2] is the difference between portfolio 5 and portfolio 2. Alphas CAPM correspond to Jensen's alphas calculated with CAPM, and Alphas FF correspond to the alphas calculated with the three-factor model. Newey-West t-statistics are reported in brackets, and p-values are reported in parentheses. * denotes significance at the 10% level, and ** denotes significance at the 5% level.

	Short-term horizon: 2 to 4 days				Short-term horizon: 4 to 8 days				Long-term horizon: more than 8 days			
Quintiles	Returns	σ	Alphas CAPM	Alphas FF	Returns	σ	Alphas CAPM	Alphas FF	Returns	σ	Alphas CAPM	Alphas FF
1	-0.09	0.38	-0.06 [-11.11] (0.00)	-0.06 [-12.11] (0.00)	-0.03	0.34	-0.02 [-4.49] (0.00)	-0.02 [-4.54] (0.00)	1.04	3.46	0.20 [3.06] (0.00)	0.12 [1.98] (0.05)
2	-0.15	0.51	-0.11 [-14.27] (0.00)	-0.11 [-14.64] (0.00)	-0.04	0.49	-0.03 [-8.53] (0.00)	-0.03 [-8.55] (0.00)	1.28	4.08	0.35 [7.69] (0.00)	0.32 [6.39] (0.00)
3	-0.26	0.66	-0.23 [-15.31] (0.00)	-0.21 [-15.49] (0.00)	-0.07	0.62	-0.06 [-10.13] (0.00)	-0.06 [-9.98] (0.00)	1.44	4.83	0.42 [7.38] (0.00)	0.43 [8.21] (0.00)
4	-0.47	0.86	-0.42 [-13.47] (0.00)	-0.41 [-13.55] (0.00)	-0.15	0.83	-0.13 [-10.90] (0.00)	-0.13 [-11.13] (0.00)	1.74	6.05	0.60 [6.20] (0.00)	0.67 [7.36] (0.00)
5	-1.04	1.23	-0.99 [-13.08] (0.00)	-0.97 [-13.20] (0.00)	-0.32	0.97	-0.31 [-14.35] (0.00)	-0.30 [-14.79] (0.00)	2.34	8.68	1.00 [4.58] (0.00)	1.07 [6.30] (0.00)
5-1	-0.96** [-12.34] (0.00)		-0.93** [-12.50] (0.00)	-0.90** [-12.61] (0.00)	-0.30** [-12.20] (0.00)		-0.29** [-13.33] (0.00)	-0.28** [-13.64] (0.00)	1.30** [3.86] (0.00)		0.80** [3.12] (0.00)	0.95** [4.89] (0.00)
5-2	-0.90** [-12.23] (0.00)		-0.88** [-12.33] (0.00)	-0.85** [-12.42] (0.00)	-0.28** [-12.37] (0.00)		-0.28** [-12.91] (0.00)	-0.27** [-13.41] (0.00)	1.06** [3.54] (0.00)		0.65** [2.83] (0.00)	0.75** [4.27] (0.00)

Table 6: Three-level MRA results. Haar wavelet.

The tables display MRA results for four groups of investment horizons: 2- to 4-day horizons, 4- to 8-day horizons, 8- to 16-day horizons and long-term horizons, from July 1963 to December 2009. Quintile 1 corresponds to the lowest idiosyncratic risk portfolio, and quintile 5 corresponds to the highest idiosyncratic risk. Returns and standard deviation (σ) are reported as percentages. [5-1] is the difference between portfolio 5 and portfolio 1, and [5-2] is the difference between portfolio 5 and portfolio 2. Alphas CAPM correspond to Jensen's alphas calculated with CAPM, and Alphas FF correspond to the alphas calculated with the three-factor model. Newey-West t-statistics are reported in brackets, and p-values are reported in parentheses. * denotes significance at the 10% level, and ** denotes significance at the 5% level.

	Short-term horizon: 2 to 4 days				Short-term horizon: 4 to 8 days				Short-term horizon: 8 to 16 days				Long term horizon: more than 16 days			
Quintiles	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**
1	-0.09	0.38	-0.06 [-11.11] (0.00)	-0.06 [-12.11] (0.00)	-0.03	0.34	-0.02 [-4.49] (0.00)	-0.02 [-4.54] (0.00)	-0.01	0.42	0.00 [-0.60] (0.55)	0.00 [-0.53] (0.60)	1.00	3.29	0.17 [2.41] (0.02)	0.10 [1.46] (0.14)
2	-0.15	0.51	-0.11 [-14.27] (0.00)	-0.11 [-14.64] (0.00)	-0.04	0.49	-0.03 [-8.53] (0.00)	-0.03 [-8.55] (0.00)	-0.03	0.55	-0.02 [-4.45] (0.00)	0.02 [-4.49] (0.00)	1.32	4.00	0.39 [8.13] (0.00)	0.34 [7.22] (0.00)
3	-0.26	0.66	-0.23 [-15.31] (0.00)	-0.21 [-15.49] (0.00)	-0.07	0.62	-0.06 [-10.13] (0.00)	-0.06 [-9.98] (0.00)	-0.04	0.71	-0.03 [-4.57] (0.00)	0.03 [-4.59] (0.00)	1.54	4.88	0.50 [8.99] (0.00)	0.50 [9.19] (0.00)
4	-0.47	0.86	-0.42 [-13.47] (0.00)	-0.41 [-13.55] (0.00)	-0.15	0.83	-0.13 [-10.90] (0.00)	-0.13 [-11.13] (0.00)	-0.08	0.94	-0.07 [-6.78] (0.00)	-0.07 [-7.29] (0.00)	1.91	6.30	0.72 [6.57] (0.00)	0.81 [8.29] (0.00)
5	-1.04	1.23	-0.99 [-13.08] (0.00)	-0.97 [-13.20] (0.00)	-0.32	0.97	-0.31 [-14.35] (0.00)	-0.30 [-14.79] (0.00)	-0.25	1.30	-0.24 [-11.50] (0.00)	-0.24 [-12.22] (0.00)	4.05	26.45	2.29 [3.03] (0.00)	1.74 [4.38] (0.00)
5-1	-0.96** [-12.34] (0.00)		-0.93** [-12.50] (0.00)	-0.90** [-12.61] (0.00)	-0.30** [-12.20] (0.00)		-0.29** [-13.33] (0.00)	-0.28** [-13.64] (0.00)	-0.24** [-9.03] (0.00)		-0.24** [-9.72] (0.00)	-0.24** [-10.34] (0.00)	3.05** [2.59] (0.01)		2.12** [2.73] (0.01)	1.64** [4.11] (0.00)
5-2	-0.90** [-12.23] (0.00)		-0.88** [-12.33] (0.00)	-0.85** [-12.42] (0.00)	-0.28** [-12.37] (0.00)		-0.28** [-12.91] (0.00)	-0.27** [-13.41] (0.00)	-0.22** [-9.31] (0.00)		-0.22** [-9.65] (0.00)	-0.22** [-10.34] (0.00)	2.73** [2.38] (0.02)		1.90** [2.53] (0.01)	1.40** [3.68] (0.00)

that the lower returns in the last quintile are explained by stocks offering a larger probability of extreme values.

To disprove this alternative explanation, we introduce a coskewness factor into the asset pricing model used to estimate idiosyncratic volatility. Using all stocks with more than 220 daily observations available, we calculate a daily coskewness factor. For each stock and year, we calculate the Harvey and Siddique (2000) coskewness measure,

$$Cosk_i = \frac{E(\varepsilon_{i,t+1}\varepsilon_{M,t+1}^2)}{\sqrt{E(\varepsilon_{i,t+1}^2)}E(\varepsilon_{M,t+1}^2)}, \qquad [7]$$

where for each stock, $\varepsilon_{i,t+1}$ are the residuals from the regression of the excess return on the contemporaneous market excess return, and $\varepsilon_{M,t+1}$ are the residuals of the excess market return over its mean.

Then, we sort the stocks into three portfolios by dividing them at 30% and 70% of the stocks and consider the two extreme portfolios. The factor value-weighted returns are calculated for the next day as the difference between the return on the lowest coskewness portfolio and the highest coskewness portfolio. The procedure is repeated by rolling the initial window by one day. Despite the promising elements that lead us to introduce the coskewness factor into our analysis, Tables 7 to 9 provide evidence against its relevance in explaining the existence of the puzzle for short-term investors. Introducing the factor has virtually no impact on D1's [5-1] returns, which are equal to -0.96% and are significant with or without coskewness. Decreases in differences are also meaningless for D2 and D3: from a significant -0.30% to a significant 0.28% for D2 and from a significant 0.24% to a significant -0.21% for D3. Furthermore, controlling for risk, CAPM and FF factors do not alter the results, and linearity in the decrease in returns is again observed. Moreover, the drop in returns for the fifth portfolio remains unchanged so that coskewness can be ruled out as an explanation for that phenomenon.

Table 7: One-level MRA results. Haar wavelet. Coskewness Factor

The tables display MRA results for two groups of investment horizons: short-term and long-term horizons, from July 1963 to December 2009. Quintile 1 corresponds to the lowest idiosyncratic risk portfolio, and quintile 5 corresponds to the highest idiosyncratic risk. Returns and standard deviation (σ) are reported as percentages. [5-1] is the difference between portfolio 5 and portfolio 1, and [5-2] is the difference between portfolio 5 and portfolio 2. Alphas CAPM correspond to Jensen's alphas calculated with CAPM, and Alphas FF correspond to the alphas calculated with the three-factor model, Newey-West t-statistics are reported in brackets, and p-values are reported in parentheses. * denotes significance at the 10% level, and ** denotes significance at the 5% level.

	Short-term horizon: 2 to 4 days				Long-term horizon: more than 4 days			
Quintiles	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**
1	-0.09	0.38	-0.06 [-11.55] (0.00)	-0.06 [-12.30] (0.00)	0.99	3.52	0.20 [2.86] (0.00)	0.11 [1.84] (0.07)
2	-0.15	0.52	-0.12 [-14.83] (0.00)	-0.12 [-15.42] (0.00)	1.23	4.25	0.34 [6.32] (0.00)	0.29 [6.06] (0.00)
3	-0.26	0.65	-0.22 [-15.44] (0.00)	-0.21 [-15.31] (0.00)	1.43	5.35	0.42 [6.07] (0.00)	0.44 [6.72] (0.00)
4	-0.47	0.86	-0.42 [-13.32] (0.00)	-0.40 [-13.42] (0.00)	1.66	7.03	0.51 [3.77] (0.00)	0.59 [5.50] (0.00)
5	-1.05	1.21	-1.00 [-13.01] (0.00)	-0.97 [-13.06] (0.00)	1.88	8.98	0.61 [2.84] (0.00)	0.70 [3.87] (0.00)
5-1	-0.96** [-12.30] (0.00)		-0.94** [-12.43] (0.00)	-0.91** [-12.47] (0.00)	0.89** [2.63] (0.01)		0.41 [1.58] (0.11)	0.59** [2.72] (0.01)
5-2	-0.90** [-12.13] (0.00)		-0.88** [-12.20] (0.00)	-0.85** [-12.23] (0.00)	0.65** [2.24] (0.03)		0.27 [1.19] (0.23)	0.41** [2.21] (0.03)

Similarly, we introduce an illiquidity factor because the literature has consistently identified it as a possible cause for the puzzle. Ang *et al*. (2006) control for both illiquidity and coskewness separately, but their approach does not include controlling for both simultaneously. We use Amihud's illiquidity measure, defined in equation [8], to build a factor for illiquidity:

$$Amih_t^i = {|ret_t^i|}/{Vol_t^i}. \qquad [8]$$

Table 8: Two-level MRA results. Haar wavelet. Coskewness Factor

The tables display MRA results for three groups of investment horizons: 2- to 4-day horizons, 4- to 8-day horizons and long-term horizons, from July 1963 to December 2009. Quintile 1 corresponds to the lowest idiosyncratic risk portfolio, and quintile 5 corresponds to the highest idiosyncratic risk. Returns and standard deviation (σ) are reported as percentages. [5-1] is the difference between portfolio 5 and portfolio 1, and [5-2] is the difference between portfolio 5 and portfolio 2. Alphas CAPM correspond to Jensen's alphas calculated with CAPM, and Alphas FF correspond to the alphas calculated with the three-factor model, Newey-West t-statistics are reported in brackets, and p-values are reported in parentheses. * denotes significance at the 10% level, and ** denotes significance at the 5% level.

	Short-term horizon: 2 to 4 days				Short-term horizon: 4 to 8 days				Long-term horizon: more than 8 days			
Quintiles	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**
			-0.06	-0.06			-0.02	-0.02			0.24	0.17
1	-0.09	0.38	[-11.55]	[-12.30]	-0.03	0.35	[-4.45]	[-4.50]	1.06	3.46	[3.89]	[3.17]
			(0.00)	(0.00)			(0.00)	(0.00)			(0.00)	(0.00)
			-0.12	-0.12			-0.03	-0.03			0.38	0.34
2	-0.15	0.52	[-14.83]	[-15.42]	-0.05	0.49	[-9.14]	[-9.08]	1.29	4.12	[8.07]	[7.06]
			(0.00)	(0.00)			(0.00)	(0.00)			(0.00)	(0.00)
			-0.22	-0.21			-0.06	-0.06			0.44	0.45
3	-0.26	0.65	[-15.44]	[-15.31]	-0.08	0.62	[-10.80]	[-10.62]	1.45	4.94	[8.12]	[8.88]
			(0.00)	(0.00)			(0.00)	(0.00)			(0.00)	(0.00)
			-0.42	-0.40			-0.13	-0.13			0.57	0.63
4	-0.47	0.86	[-13.32]	[-13.42]	-0.14	0.82	[-10.20]	[-10.41]	1.69	6.15	[5.66]	[6.57]
			(0.00)	(0.00)			(0.00)	(0.00)			(0.00)	(0.00)
			-1.00	-0.97			-0.29	-0.29			1.09	1.15
5	-1.05	1.21	[-13.01]	[-13.06]	-0.31	0.95	[-13.68]	[-13.97]	2.38	8.75	[5.07]	[6.24]
			(0.00)	(0.00)			(0.00)	(0.00)			(0.00)	(0.00)
	-0.96**		-0.94**	-0.91**	-0.28**		-0.27**	-0.27**	1.32**		0.85**	0.98**
5-1	[-12.30]		[-12.43]	[-12.47]	[-11.81]		[-12.59]	[-12.97]	[4.01]		[3.49]	[4.84]
	(0.00)		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)
	-0.90**		-0.88**	-0.85**	-0.26**		-0.26**	-0.26**	1.09**		0.71**	0.81**
5-2	[-12.13]		[-12.20]	[-12.23]	[-11.68]		[-12.09]	[-12.37]	[3.65]		[3.14]	[4.13]
	(0.00)		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)

Table 9: Three-level MRA results. Haar wavelet. Coskewness Factor

The tables display MRA results for four groups of investment horizons; 2- to 4-day horizons, 4- to 8-day horizons, 8- to 16-day horizons and long-term horizons, from July 1963 to December 2009. Quintile 1 corresponds to the lowest idiosyncratic risk portfolio, and quintile 5 corresponds to the highest idiosyncratic risk. Returns and standard deviation (σ) are reported as percentages. [5-1] is the difference between portfolio 5 and portfolio 1, and [5-2] is the difference between portfolio 5 and portfolio 2. Alphas CAPM correspond to Jensen's alphas calculated with CAPM, and Alphas FF correspond to the alphas calculated with the three-factor model. Newey-West t-statistics are reported in brackets, and p-values are reported in parentheses. * denotes significance at the 10% level, and ** denotes significance at the 5% level.

	Short-term horizon: 2 to 4 days				Short-term horizon: 4 to 8 days				Short-term horizon: 8 to 16 days				Long-term horizon: more than 16 days			
Quintiles	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**
1	-0.09	0.38	-0.06 [-11.55] (0.00)	-0.06 [-12.30] (0.00)	-0.03	0.35	-0.02 [-4.45] (0.00)	-0.02 [-4.50] (0.00)	-0.01	0.43	-0.01 [-0.81] (0.42)	0.00 [-0.73] (0.46)	1.03	3.39	0.21 [3.43] (0.00)	0.14 [2.71] (0.01)
2	-0.15	0.52	-0.12 [-14.83] (0.00)	-0.12 [-15.42] (0.00)	-0.05	0.49	-0.03 [-9.14] (0.00)	-0.03 [-9.08] (0.00)	-0.02	0.56	-0.02 [-3.84] (0.00)	-0.02 [-3.87] (0.00)	1.33	4.02	0.41 [7.98] (0.00)	0.38 [7.27] (0.00)
3	-0.26	0.65	-0.22 [-15.44] (0.00)	-0.21 [-15.31] (0.00)	-0.08	0.62	-0.06 [-10.80] (0.00)	-0.06 [-10.62] (0.00)	-0.04	0.70	-0.04 [-5.19] (0.00)	-0.04 [-5.25] (0.00)	1.48	4.82	0.46 [8.48] (0.00)	0.47 [8.81] (0.00)
4	-0.47	0.86	-0.42 [-13.32] (0.00)	-0.40 [-13.42] (0.00)	-0.14	0.82	-0.13 [-10.20] (0.00)	-0.13 [-10.41] (0.00)	-0.08	0.92	-0.07 [-6.77] (0.00)	-0.08 [-7.19] (0.00)	1.77	5.94	0.64 [6.51] (0.00)	0.71 [8.19] (0.00)
5	-1.05	1.21	-1.00 [-13.01] (0.00)	-0.97 [-13.06] (0.00)	-0.31	0.95	-0.29 [-13.68] (0.00)	-0.29 [-13.97] (0.00)	-0.22	1.29	-0.22 [-10.56] (0.00)	-0.22 [-11.35] (0.00)	2.99	14.05	1.53 [3.88] (0.00)	1.34 [5.03] (0.00)
5-1	-0.96** [-12.30] (0.00)		-0.94** [-12.43] (0.00)	-0.91** [-12.47] (0.00)	-0.28** [-11.81] (0.00)		-0.27** [-12.59] (0.00)	-0.27** [-12.97] (0.00)	-0.21** [-8.08] (0.00)		-0.21** [-8.62] (0.00)	-0.22** [-9.30] (0.00)	1.96** [3.18] (0.00)		1.32** [3.14] (0.00)	1.20** [4.16 `(0.00)
5-2	-0.90** [-12.13] (0.00)		-0.88** [-12.20] (0.00)	-0.85** [-12.23] (0.00)	-0.26** [-11.68] (0.00)		-0.26** [-12.09] (0.00)	-0.26** [-12.37] (0.00)	-0.20** [-8.44] (0.00)		-0.19** [-8.73] (0.00)	-0.20** [-9.45] (0.00)	1.66** [2.87] (0.00)		1.12** [2.85] (0.00)	0.96** [3.74] (0.00)

For all stocks with more than 200 observations each year, Amihud's measure is calculated. Stocks are then sorted according to annual Amihud's mean measure and are assigned to one of three portfolios. The factor value-weighted returns are calculated for the next day as the difference between the return on the highest illiquidity portfolio and the lowest illiquidity portfolio. Here, the divisions are introduced so that each of the portfolios contains 30% of the stocks. The procedure is repeated by rolling the initial window by one day.

Table 10: One-level MRA results. Haar wavelet. Illiquidity and Coskewness Factors

The tables display MRA results for two groups of investment horizons: short-term and long-term horizons, from July 1963 to December 2009. Quintile 1 corresponds to the lowest idiosyncratic risk portfolio, and quintile 5 corresponds to the highest idiosyncratic risk. Returns and standard deviation (σ) are reported as percentages. [5-1] is the difference between portfolio 5 and portfolio 1, and [5-2] is the difference between portfolio 5 and portfolio 2. Alphas CAPM correspond to Jensen's alphas calculated with CAPM, and Alphas FF correspond to the alphas calculated with the three-factor model. Newey-West t-statistics are reported in brackets, and p-values are reported in parentheses. * denotes significance at the 10% level, and ** denotes significance at the 5% level.

	Short-term horizon: 2 to 4 days				Long-term horizon: more than 4 days			
Quintiles	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**
1	-0.09	0.39	-0.06 [-11.13] (0.00)	-0.06 [-11.80] (0.00)	1.02	3.52	0.22 [3.17] (0.00)	0.14 [2.28] (0.02)
2	-0.16	0.51	-0.12 [-14.03] (0.00)	-0.12 [-14.95] (0.00)	1.21	4.26	0.31 [6.20] (0.00)	0.27 [6.01] (0.00)
3	-0.26	0.64	-0.22 [-15.42] (0.00)	-0.21 [-15.28] (0.00)	1.41	5.29	0.40 [5.61] (0.00)	0.42 [5.94] (0.00)
4	-0.45	0.85	-0.40 [-13.74] (0.00)	-0.39 [-13.82] (0.00)	1.65	6.94	0.51 [3.80] (0.00)	0.59 [5.62] (0.00)
5	-1.03	1.22	-0.98 [-12.88] (0.00)	-0.95 [-12.92] (0.00)	1.84	8.94	0.57 [2.72] (0.01)	0.66 [3.68] (0.00)
5-1	-0.94** [-12.21] (0.00)		-0.92** [-12.31] (0.00)	-0.89** [-12.35] (0.00)	0.82** [2.48] (0.01)		0.35 [1.37] (0.17)	0.52** [2.46] (0.01)
5-2	-0.87** [-12.00] (0.00)		-0.86** [-12.05] (0.00)	-0.83** [-12.06] (0.00)	0.63** [2.22] (0.03)		0.26 [1.16] (0.25)	0.39** [2.12] (0.03)

Table 11: Two-level MRA results. Haar wavelet. Illiquidity and Coskewness Factors

The tables display MRA results for three groups of investment horizons: 2- to 4-day horizons, 4- to 8-day horizons and long-term horizons, from July 1963 to December 2009. Quintile 1 corresponds to the lowest idiosyncratic risk portfolio, and quintile 5 corresponds to the highest idiosyncratic risk. Returns and standard deviation (σ) are reported as percentages. [5-1] is the difference between portfolio 5 and portfolio 1, and [5-2] is the difference between portfolio 5 and portfolio 2. Alphas CAPM correspond to Jensen's alphas calculated with CAPM, and Alphas FF correspond to the alphas calculated with the three-factor model. Newey-West t-statistics are reported in brackets, and p-values are reported in parentheses. * denotes significance at the 10% level, and ** denotes significance at the 5% level.

	Short-term horizon: 2 to 4 days				Short-term horizon: 4 to 8 days				Long-term horizon: more than 8 days			
Quintiles	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**
1	-0.09	0.39	-0.06 [-11.13] (0.00)	-0.06 [-11.80] (0.00)	-0.03	0.35	-0.02 [-3.62] (0.00)	-0.02 [-3.58] (0.00)	1.12	3.49	0.29 [4.89] (0.00)	0.22 [4.29] (0.00)
2	-0.16	0.51	-0.12 [-14.03] (0.00)	-0.12 [-14.95] (0.00)	-0.05	0.49	-0.03 [-9.10] (0.00)	-0.03 [-8.96] (0.00)	1.27	4.10	0.35 [7.16] (0.00)	0.33 [6.58] (0.00)
3	-0.26	0.64	-0.22 [-15.42] (0.00)	-0.21 [-15.28] (0.00)	-0.08	0.62	-0.06 [-10.30] (0.00)	-0.06 [-10.20] (0.00)	1.44	4.86	0.44 [8.38] (0.00)	0.45 [8.70] (0.00)
4	-0.45	0.85	-0.40 [-13.74] (0.00)	-0.39 [-13.82] (0.00)	-0.14	0.81	-0.12 [-10.41] (0.00)	-0.12 [-10.62] (0.00)	1.59	5.98	0.48 [5.13] (0.00)	0.55 [6.16] (0.00)
5	-1.03	1.22	-0.98 [-12.88] (0.00)	-0.95 [-12.92] (0.00)	-0.30	0.97	-0.28 [-13.40] (0.00)	-0.28 [-13.60] (0.00)	2.24	8.41	0.98 [4.84] (0.00)	1.02 [6.11] (0.00)
5-1	-0.94** [-12.21] (0.00)		-0.92** [-12.31] (0.00)	-0.89** [-12.35] (0.00)	-0.27** [-11.42] (0.00)		-0.26** [-12.27] (0.00)	-0.26** [-12.55] (0.00)	1.12** [3.64] (0.00)		0.69** [2.99] (0.00)	0.80** [4.34] (0.00)
5-2	-0.87** [-12.00] (0.00)		-0.86** [-12.05] (0.00)	-0.83** [-12.06] (0.00)	-0.25** [-11.00] (0.00)		-0.25** [-11.46] (0.00)	-0.25** [-11.66] (0.00)	0.97** [3.46] (0.00)		0.63** [2.85] (0.00)	0.69** [3.77] (0.00)

Table 12: Three-level MRA results. Haar wavelet. Illiquidity and Coskewness Factors

The tables display MRA results for four groups of investment horizons: 2- to 4-day horizons, 4- to 8-day horizons, 8- to 16-day horizons and long-term horizons, from July 1963 to December 2009. Quintile 1 corresponds to the lowest idiosyncratic risk portfolio, and quintile 5 corresponds to the highest idiosyncratic risk. Returns and standard deviation (σ) are reported as percentages. [5-1] is the difference between portfolio 5 and portfolio 1, and [5-2] is the difference between portfolio 5 and portfolio 2. Alphas CAPM correspond to Jensen's alphas calculated with CAPM, and Alphas FF correspond to the alphas calculated with the three-factor model. Newey-West t-statistics are reported in brackets, and p-values are reported in parentheses. * denotes significance at the 10% level, and ** denotes significance at the 5% level.

	Short-term horizon: 2 to 4 days				Short-term horizon: 4 to 8 days				Short-term horizon: 8 to 16 days				Long-term horizon: more than 16 days			
Quintiles	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**	**Returns**	**σ**	**Alphas CAPM**	**Alphas FF**
1	-0.09	0.39	-0.06 [-11.13] (0.00)	-0.06 [-11.80] (0.00)	-0.03	0.35	-0.02 [-3.62] (0.00)	-0.02 [-3.58] (0.00)	-0.01	0.43	-0.01 [-1.46] (0.15)	-0.01 [-1.41] (0.16)	1.04	3.36	0.22 [3.21] (0.00)	0.16 [2.63] (0.01)
2	-0.16	0.51	-0.12 [-14.03] (0.00)	-0.12 [-14.95] (0.00)	-0.05	0.49	-0.03 [-9.10] (0.00)	-0.03 [-8.96] (0.00)	-0.03	0.55	-0.02 [-3.99] (0.00)	-0.02 [-4.01] (0.00)	1.33	4.05	0.41 [7.86] (0.00)	0.36 [6.75] (0.00)
3	-0.26	0.64	-0.22 [-15.42] (0.00)	-0.21 [-15.28] (0.00)	-0.08	0.62	-0.06 [-10.30] (0.00)	-0.06 [-10.20] (0.00)	-0.04	0.69	-0.03 [-4.84] (0.00)	-0.03 [-4.84] (0.00)	1.45	4.91	0.42 [7.77] (0.00)	0.44 [7.87] (0.00)
4	-0.45	0.85	-0.40 [-13.74] (0.00)	-0.39 [-13.82] (0.00)	-0.14	0.81	-0.12 [-10.41] (0.00)	-0.12 [-10.62] (0.00)	-0.07	0.90	-0.06 [-5.68] (0.00)	-0.07 [-5.99] (0.00)	1.89	6.22	0.74 [6.54] (0.00)	0.81 [8.49] (0.00)
5	-1.03	1.22	-0.98 [-12.88] (0.00)	-0.95 [-12.92] (0.00)	-0.30	0.97	-0.28 [-13.40] (0.00)	-0.28 [-13.60] (0.00)	-0.20	1.24	-0.19 [-10.17] (0.00)	-0.19 [-10.84] (0.00)	3.94	25.89	2.25 [3.06] (0.00)	1.69 [4.33] (0.00)
5-1	-0.94** [-12.21] (0.00)		-0.92** [-12.31] (0.00)	-0.89** [-12.35] (0.00)	-0.27** [-11.42] (0.00)		-0.26** [-12.27] (0.00)	-0.26** [-12.55] (0.00)	-0.19** [-7.47] (0.00)		-0.18** [-8.02] (0.00)	-0.18** [-8.56] (0.00)	2.90** [2.55] (0.01)		2.03** [2.71] (0.01)	1.53** [3.91] (0.00)
5-2	-0.87** [-12.00] (0.00)		-0.86** [-12.05] (0.00)	-0.83** [-12.06] (0.00)	-0.25** [-11.00] (0.00)		-0.25** [-11.46] (0.00)	-0.25** [-11.66] (0.00)	-0.17** [-7.72] (0.00)		-0.17** [-8.09] (0.00)	-0.17** [-8.67] (0.00)	2.61** [2.36] (0.02)		1.84** [2.53] (0.01)	1.33** [3.52] (0.00)

As shown in Tables 10 to 12, the inclusion of both factors in the analysis does not account for the puzzle. As in the previous case, decreases in [5-1] returns are small, and the negative relationship is still significant for all short-term investors.

Conclusions

For classical asset pricing theory, the negative link between idiosyncratic risk and expected returns established by Ang *et al*. (2006, 2009) is challenging. We believe that in order to study the anomaly it is necessary to consider the effect of empirical issues such as fat tails in return distributions and theoretical reasons behind these issues. In particular, we propose a heterogeneous market framework that provides a possible theoretical explanation for the puzzle and seems to better fit the empirical data. Our main conclusion is that the puzzle reported above disappears for long-term horizons and holds for the short-term investors. Our findings note the relevance of the puzzle only for investors with short-run horizons compared with those with long-run horizons. This result holds when controlling for different wavelet families and different idiosyncratic risk estimators, including illiquidity and coskewness effects. Moreover, adding several definitions of short-term investors provides corroborating evidence for our hypothesis that the puzzle is related to investors' time horizon. This is because the link between idiosyncratic risk and returns becomes weaker as we increase the number of short-term investor groups.

Chapter III: Idiosyncratic volatility anomaly: corporate investment or investors mispricing?

3.1. Introduction

The concept of diversification rules out any predictive power of idiosyncratic risk over expected returns and is one of the strongest ones in asset pricing. Therefore, the fact that the portfolios with highest idiosyncratic risk levels yield significantly lower returns than those with the lowest levels of it came as a puzzling surprise (Ang *et al.,* 2006 and 2009). Although this observation was initially contested in papers such as Bali and Cakici (2008) and Fu (2009) it has ultimately gained full recognition and became known as the idiosyncratic volatility anomaly. At first sight, the anomaly constitutes a challenge either to the idea of diversification or to the models used to estimate idiosyncratic risk. But given that in contradiction to the anomaly, under-diversification models such as Merton (1987) anticipate a positive relationship between idiosyncratic risk and expected returns, the accuracy of the CAPM or the Fama and French (1993) model seems to be the option to go with. Indeed, idiosyncratic risk is always estimated as a residual from a particular asset pricing model so that if the model is inaccurate then the measure of idiosyncratic risk could be catching more information that it should. In this sense, it seems plausible that the anomaly arises due to the lack of relevant controls related to stock returns.

Surprisingly, most of the literature leaves this option aside and approaches the anomaly through more complex rationales including investor preferences, market microstructure issues, arbitrage costs and investor irrationality. Papers such as Kapadia (2006) and Boyer *et al*., (2010) who show investors tilt towards high firm-specific volatility stocks whose returns distributions offer desirable features like positive skewness or lottery-like payoffs (Bali *et al*., 2011) explore the effect of investor preferences on the anomaly. Microstructure issues such as returns reversals or trading non-synchronicity are linked to the anomaly by Huang *et at.*, (2010) and Han and

Lesmond, (2011) respectively but are refuted by Chen *et al.,* (2012a) showing that the debate is sound and still developing. Also, papers related to arbitrage costs argue that idiosyncratic risk determines arbitrage cost so that the anomaly is not arbitraged away because high idiosyncratic risk stocks are difficult to short (Boehme *et al*., 2009; Au *et al*., 2009; Cao, 2009; Duan *et al*., 2010). Finally, some authors relate the anomaly to investor irrationality; Gao *et al.,* (2012) argue the relationship between idiosyncratic volatility and expected returns depends on investor sentiment so that the anomaly is only observed during times following high investor sentiment periods. In contrast, Jiang *et al.,* (2009) refute the hypothesis that accruals anomaly (based on the irrationality of investors when assessing cash flows information content) explains the idiosyncratic volatility anomaly.

In this paper we follow the idea that the idiosyncratic volatility anomaly might be observed given the lack of controls related to stock returns and we argue these controls should account for managerial decisions within the firm since these should have a large idiosyncratic effect on firm's stock returns. In particular, in this paper we contribute to the discussion by studying the effect of corporate investment on the observation of the idiosyncratic volatility anomaly. Our rationale is threefold. First, firm investment is mostly idiosyncratic given that each manager faces very unique conditions when he decides which investment projects are to be undertaken. Second, given that investments are associated to higher uncertainty of future cash flows, investment level should significantly modify firms overall risk and, in turn, its idiosyncratic risk component. Third, valuation theory offers a framework where expected returns, profitability and investment are theoretically linked and where the true negative relationship between investment and expected returns arises only after controls for profitability are included. Moreover, this negative relationship should persist with independence of investor rationality (Fama and French, 2006).

Introducing the discussion of the effect of investment on the idiosyncratic volatility anomaly we pursue two objectives. On the one hand, we believe we fill a gap in the literature that has until now marginalized the corporations side from the analysis. On the other hand, we offer a working hypothesis that is broad in implications. Indeed, in addition to provide a plausible explanation for the anomaly, the hypothesis that joint controls for profitability and investment account for the anomaly with independence of investor rationality allows us to refute two recent papers that seemed to support irreconcilable hypotheses. These are, the conception that the anomaly arises purely from investor irrationality during times of high investor sentiment exposed by Gao *et al*., (2012) and, the idea that the anomaly is not linked at all to investor irrationality when coming to assess accruals information advanced by Jiang *et al.,* (2009). Furthermore, our approach allows testing a hypothesis based on rationality that should be proven false before considering any hypothesis based on irrationality.

Our results strongly support our hypothesis. On the one hand, there is a linear relationship between investment and idiosyncratic risk that supports our initial idea that investment and idiosyncratic risk should correlate positively. On the other hand, investment is by itself insufficient to explain the anomaly in the cross-sectional analysis, but considering both profitability and investment does account for the idiosyncratic risk anomaly. Moreover, these controls also proof to prevail both during times succeeding high investor sentiment and during times succeeding low investor sentiment. It seems then that the idiosyncratic volatility anomaly is not related to investor irrational expectations but to managerial decision making affecting both investment and profitability of the firms and that are not fully considered in the asset pricing model used to estimate the firm specific risk. Our results also emphasize the implications of valuation theory in terms of the literature on the idiosyncratic volatility anomaly. First, they offer an explanation to why Jiang *et al.,* (2009) fail to link investor irrationality in assessing accruals information content and the idiosyncratic risk anomaly; it is not enough to control for investment related variables such as accruals. Second, they account for the anomaly both during high and during low investor sentiment periods, refuting the argument by Gao *et al.,* (2012) that the anomaly is purely dependent on investor rationality.

The remainder of the paper is organized as follows. Section 2 gives a general discussion on the explanations provided for the empirical observation of a negative link between investment and expected returns and their relevance for our approach to the idiosyncratic volatility anomaly. Data description, methodology and preliminary evidence of the anomaly in our sample are discussed in Section 3. Section 4 describes our empirical findings over the puzzle after controlling by corporate variables. Section 5 concludes.

3.2. Corporate investment and the cross section of stock returns

Our hypothesis is based on the idea that there is an effect non-captured in the idiosyncratic risk measure. Therefore, two conditions are necessary; first, the missing effect should be negatively correlated with expected returns and second, this missing effect should be positively related to idiosyncratic risk. We argue investment is a good candidate for this missing variable because it fulfills both necessary conditions. On the one hand, investment and idiosyncratic risk should be positively related because (i) investment results from a decision making process conditioned by purely firm-specific variables such as managerial aversion to risk or financing conditions faced by the firm and, (ii) intuitively, investment should increase idiosyncratic volatility because it increases uncertainty over future cash flows. Also, papers such as Chen *et al*., (2012b) support this idea by showing that measures related to managerial activity such as discretionary accruals reduce information quality and this, in turn, induces higher idiosyncratic return volatility. Therefore, managerial decisions on investment levels and the nature of the projects undertaken by firms could have an impact on the idiosyncratic volatility anomaly.

On the other hand, there are at least two theoretical approaches justifying a negative link between investment and expected returns. The first one is related to the accruals anomaly and implies that this negative link is driven by investor mispricing of past accounting information. The second one is based on the valuation theory and implies that the negative link between investment and expected returns arises analytically from a valuation equation and is totally independent from investor rationality. The differences and similarities between these two approaches are of major importance for our argument to be proven right. Therefore, in this section we develop both theoretical arguments and show that they are closely related in terms of their empirical implications. By doing so, we also emphasize that the relevance of investor irrationality changes dramatically from one approach to the other. Finally, in the last part of the section we highlight what type of empirical results would support our hypothesis that idiosyncratic volatility anomaly is related to corporate investment and profitability and not related either to investor expectations, as implied by Jiang *et al.,* (2009), or to investor sentiment, as implied by Gao *et al.,* (2012).

From the literature documenting a negative link between expected returns and investment arising from investor mispricing, accruals literature might be the most influential one.[17] After Sloan (1996) seminal paper, a large body of evidence has been developed around the idea that investors underreact to cash-flows information content. Most of the accruals literature follows the argument that investors are unable to correctly differentiate earnings from accruals which, since they are less persistent, lead investors to overvalue firms with larger accruals. This turns into a negative relationship between accruals and expected returns, known as the accruals anomaly, as the largest the accruals component of cash flows, the largest the drop in the stock price when realized earnings differ from investors' expectations (e.g. Sloan, 1996, or Richardson *et al.,* 2005, among others).

Building on the accruals anomaly, authors such as Cooper *et al.*, (2008) provide evidence in favor of a larger anomaly related to the mispricing of investment growth in general because firm-specific asset growth dominates other variables usually negatively correlated to the cross-section of stock returns such as accruals, sales growth and growth in capital investment. Authors conclude that their results signal strong empirical evidence on the inability of financial markets to price both investment and disinvestment activities and not only some of its components. Similarly to the accruals argument the negative relationship between investment and expected returns in this context results from investors' lack of sophistication in the sense that they overreact to past accounting information and therefore misprice the stocks.

A particularly relevant critique to the irrationality idea underlying the accruals anomaly is offered by Fama and French (2006) that argue the negative link between investment and expected returns arises analytically after controlling for profitability and without the need to assume irrational expectations[18]. Based on the valuation theory, Fama and French argue the observation of the accrual anomaly could arise from the dividend discount model in equation 1:

[17] The fact that investment and accruals are closely related would become clear at the end of this section.

[18] There are additional approaches for the empirical observation implied by the accruals anomaly that do not consider mispricing and irrationality as the source of the negative link between investment and expected returns. Noteworthy examples are Fairfield et al., (2003) and Dechow (2008) suggesting accruals are measures of invested capital and relating the negative association between accruals and expected returns to diminishing marginal returns to investment.

$$M_t = \sum_{\tau=1}^{\infty} E(D_{t+\tau})/(1+r)^{\tau}, \qquad [1]$$

where M_t is a share market price at time t, $E(D_{t+\tau})$ is the expected dividend in period t+τ, and r is the internal rate of returns on expected dividends. Considering clean surplus accounting and dividing by book equity the model can be written as equation 2:

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t}, \qquad [2]$$

where Y_t is equity earnings per share at time t and $dB_t = B_t - B_{t-1}$ is change in book equity per share, B_t.[19]

Then, assuming the internal rate of returns on expected dividends, r, is approximately equivalent to the long-term average expected stock return, and fixing M_t/B_t and expected earnings to book equity, firms with higher expected equity investment, $dB_{t+\tau}$, have lower expected returns. Also, controlling for M_t/B_t and expected growth in book equity, more profitable firms have higher expected returns (Fama and French, 2006). This is to say there are two interrelated effects, the profitability effect and the investment effect, that should not be treated in isolation. To study the relationship between any two of these variables: investment, profitability and expected returns, it becomes necessary to control for the third one. The negative relationship between investment and expected returns should be the result of controlling for profitability.

Notice that in this framework the negative relation between investment and expected returns is silent about the nature of pricing so that investors' valuation can arise both from rational or irrational expectations. The model is not able to determine if the effects of investment and profitability on average stocks returns are related to rational or irrational pricing. Seen in another way, the negative link between expected returns and investment after controlling for profitability should yield both under rational and under irrational expectations. Additional evidence in favor of this approach is provided by Chen *et al.*, (2011) that develop an asset pricing model which factors are based on profitability and investment and that performs well in explaining the cross-

[19] For a discussion on why results could hold even if the clean surplus accounting assumption is violated see Fama and French (2006).

section of stock returns and, Novy-Marx (2013) that argue current profitability predicts returns through its effect on important determinants of future stock prices such as earnings, cash-flows and payouts.

In the context of the idiosyncratic risk anomaly the discussion of the investment and profitability effects offers several interesting opportunities. On the one hand, it allows us to contribute to the debate on the role of investment in the anomaly opened by Jiang *et al.,* (2009) who conclude the accruals anomaly does not account for the idiosyncratic risk one. In this paper we include a different measure for investment and propose an alternative interpretation unrelated to investors that highlights the primordial role investment has on the idiosyncratic risk anomaly. Another interesting point is that the fact that, as shown below in this section, our hypothesis that controlling simultaneously for profitability and investment the idiosyncratic volatility could no longer be observed should hold both under rationality and under irrationality. In particular, this fact allows us to contribute to the discussion about the relevance of investor sentiment as a determinant of the idiosyncratic volatility anomaly initiated by Gao *et al*., (2012) who show the anomaly is only observed in times of high investor sentiment, concluding that the anomaly is produced by a mispricing from investors.

In terms of the results we expect it is imperative to understand that our measure for investment is clearly related to accruals (this is shown at the end of the following section). Therefore, we expect that controlling for investment would not be sufficient to account for the idiosyncratic volatility anomaly. Also, for our hypothesis to be supported two elements are required. On the one hand, joint controls for investment and profitability should result in the relationship between idiosyncratic risk and expected returns to become either non-significant or positive. On the other hand, the effectiveness of these controls should prevail both during times of high investor sentiment when irrational expectations from sentiment investors dominate and during times of low investor sentiment when the dominant expectations are arbitrageurs' rational ones.

3.3. Data, methodology and preliminary evidence

3.3.1. Data and methodology

We approach the study of the idiosyncratic volatility anomaly through two methodologies; in this section we perform a portfolio sorting methodology and in the following section we discuss the results of several Fama and Macbeth (1973) regressions where joint controls for profitability and investment are included. The study is developed using daily returns information on all non-financial common stocks (SIC codes 6000 to 6999) in the NYSE, AMEX and Nasdaq available both in CRSP and Compustat since our measures for profitability and for investment are based on balance sheet information.[20] To allow for accounting information to become public knowledge we leave a window of six months after each fiscal year end. The resulting sample dates from July 1982 to December 2009 and includes 1.127.147 firm-month observations, approximately 3.415 firms per month. For the Fama-MacBeth regressions in section 4 the sample is reduced to 1.004.965 firm-month observations by eliminating the months for which mergers and acquisitions result in a strong variation of our accounting measures.

We consider six alternative measures of profitability and only one for investment since we believe it is the most comprehensive measure that can be possibly constructed. Three of our profitability measures are based on Tobin's q as defined by Verdi (2006); the first proxy labeled "Tobin's q" is defined as:

$$Q_t = \frac{TA_t + MVEq_t - BVEq_t}{TA_t}, \qquad [3]$$

where, TA is total assets (Compustat item #6), MVEq stands for market value of equity defined as common stocks outstanding (Compustat item #25) times stock price (Compustat item #199) and BVEq is book value of equity (Compustat item #60). Using the mean stock price for each month reported in CRSP instead of the information on Compustat, we define our second proxy for profitability labeled "Tobin's q var". This measure has the advantage that it incorporates the most recent information investors have on stock prices and varies month by month. Our third proxy labeled "Tobin'q min"

[20] We only consider firms in the CRSP with more than 17 daily observations in a month.

is the minimum between these two measures and is used in order to minimize the effect of outliers.

We also consider three additional profitability measures based on Fama and French (2006); dividends per share (Compustat item #26), a dummy variable equal to one for dividend paying firms and profitability defined as in equation 4:

$$profitability_t = \frac{IBEI_t}{TA_t - L_t + A_t}, \qquad [4]$$

where, IBEI stands for income before extraordinary items (Compustat item #18), TA is total assets (Compustat item #6), L is total liabilities (Compustat item #181) and A is a term equal to balance sheet deferred taxes investment tax credit (Compustat item #35) if available minus preferred stock liquidating value (Compustat item #10) if available, or redemption value (Compustat item #56) if available, or carrying value (Compustat item #130).

Our investment proxy is asset growth defined as the growth rate of total assets (Compustat item #6) during the previous two years (Cooper *et al.,* 2008):

$$AG_t = \frac{TA_{t-1} - TA_{t-2}}{TA_{t-2}}, \qquad [5]$$

where AG stands for asset growth and TA is total assets. We do not consider any more measures of investment since any alternative accounting based definition of investment should be included in asset growth and also because this measure allows us to contrast our results with the results expected in the accruals framework. This is possible because as we show in the following lines asset growth is also a proxy for accruals.

In fact, given the extension of the literature on accruals anomaly, several definitions of accruals are available.[21] For our purposes we highlight the one provided by Richardson *et al*. (2005) who define accruals as the left-hand side of equation [6]:

$$\Delta NOA = \Delta NFO + \Delta B, \qquad [6]$$

[21] For a comprehensive review of accounting anomalies such as the investment anomaly see Richardson *et al.* (2010).

where, NOA stands for net operating assets and is derived as the difference between operating assets and operating liabilities, NFO stands for net financial obligations and is calculated as short-term debt plus long-term debt less financial assets, B stands for book value of equity and Δ denotes changes.

Note that this accruals measure is mechanically related to asset growth as shown through equations [7] to [9]. Defining asset growth as:

$$\Delta TA = \Delta OA + \Delta FA, \qquad [7]$$

where, ΔTA is change in total assets, ΔOA is change in operating assets and ΔFA is change in financial assets.

Also, a change in net operating assets can be defined as:

$$\Delta NOA = \Delta OA - \Delta OL, \qquad [8]$$

where, ΔOL stands for operating liabilities and the rest of the variables are defined as before.

Thus, from equations [7] and [8], asset growth can be written as shown in equation [9]:

$$\Delta TA = \Delta NOA + \Delta OL + \Delta FA, \qquad [9]$$

making clear that asset growth proxies for accruals. Since asset growth is a measure of investment, it is thus possible to establish the following equivalence in the context of our study: asset growth ≡ accruals ≡ investment and to link asset growth and expected returns negatively both in the context of investor irrational expectations implied by the accruals anomaly literature and in the context of the valuation theory independent from investor expectations. This fact will become relevant when several variables are included in the Fama and Macbeth regressions in the second part of this section.

3.3.2. *Preliminary evidence*

Before introducing any controls for profitability or investment we verify that the idiosyncratic volatility puzzle is observed in our sample. Each month of year t we consider firms reporting information for the previous fiscal year in Compustat. Firms are sorted monthly from the lowest to the highest level of idiosyncratic risk as defined by the standard deviation of the residuals of equation [10], ($\sigma_{\varepsilon_t^i}$).

$$R_t^i = \alpha^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \varepsilon_t^i, \qquad [10]$$

where, r_t^i is the stock return in excess of the risk free rate and $\{MKT_t, SMB_t, HML_t\}$ represent the market, size and book to market factors.[22] Then, we form quintiles and calculate monthly value weighted returns for each of them. Once the vector of monthly returns is formed for each quintile, we also calculate their Fama and French three-factor model alphas.

Table 1 summarizes the information on the idiosyncratic risk anomaly for our sample. Columns report average monthly returns, alphas and mean asset growth (all in percentages) for each quintile of idiosyncratic risk. Since stocks are organized so that the first quintile has lower idiosyncratic risk than the fifth one, we expect the [5-1] difference to be significantly negative. As expected, the idiosyncratic volatility anomaly is observed both in raw and risk adjusted returns; the [5-1] difference in monthly raw returns is equal to a significant -0.77% and is even more pronounced in risk adjusted terms. Difference in alphas is equal to -1.17% with an associated t-statistic of -4.08. In addition, we observe a pattern common to previous studies of the idiosyncratic volatility anomaly; a sharp drop in the returns is observed on the fourth or fifth quintile (Chen and Petrokva, 2011, Malagon *et al.,* 2013a and Jiang *et al.*, 2009).

In the framework of our working hypothesis that investment should increase idiosyncratic risk, Table 1 provides a major argument in favour because mean asset growth increases linearly with idiosyncratic risk. Notice also that the [5-1] difference in asset growth is significant. Although we do not perform any causality test, we argue our intuition that corporate investment and firm's idiosyncratic volatility should be related

[22] They have been obtained from Kenneth French's website
http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library html

cannot be dismissed. On the other hand, we argue the observation of the idiosyncratic risk anomaly is the joint result of the investment and the profitability effects identified by Fama and French (2006). It is therefore important to check that none of our variables accounts by itself for the anomaly. In order to test this fact, we check for the anomaly to be observed after controlling separately by investment and by profitability.

Table 1: Returns of portfolios sorted by idiosyncratic risk

This table reports monthly average returns, risk adjusted returns and mean asset growth for quintiles formed after sorting stocks according to their level of idiosyncratic risk including all non-financial (SIC codes 6000 - 6999) common stocks available jointly on CRSP and Compustat from July 1982 to December 2009 (approximately 3.415 firms per month). Quintile 1 corresponds to the portfolio having the lowest idiosyncratic risk and quintile 5 to the portfolio with highest idiosyncratic risk. Returns, Alphas and Asset Growth are reported in monthly percentage. The row [5-1] is the difference between portfolio 5 and portfolio 1. Newey-West *t*-statistics are reported in brackets. Alphas correspond to the intercept of [1].

Idiosyncratic risk quintiles	Returns	Alphas	Asset Growth
Low 1	1.26	0.30 [3.76]	0.49
2	1.17	0.08 [0.88]	0.49
3	1.26	0.07 [0.61]	1.31
4	1.09	-0.12 [-0.75]	2.95
High 5	0.49	-0.87 [-3.64]	5.82
5-1	-0.77 [-1.74]	-1.17 [-4.08]	5.31 [2.44]

To control for the investment effect, in Table 2 we first sort stocks according to investment and we form quintiles. Then, within each portfolio we sort stocks according to their idiosyncratic risk level and form quintiles again. All sorts are done from the lowest to the highest level. Once all quintiles are constructed, both monthly value weighted returns and alphas are calculated for each of the 25 resulting portfolios.[23] Since accounting information is released yearly, asset growth's measure is kept for a whole year until new information comes to the market and the position of a given firm in the sort based on it is the same from one year to the other. Both reported raw returns (Panel A) and alphas (Panel B) are in percentages. The same procedure is followed to

[23] Alphas refer here to alphas from a Fama and French three factor model where each portfolio is treated as an asset to be priced.

Table 2: Raw and risk-adjusted returns by levels of asset growth

This table reports both raw and risk adjusted returns for 25 quintile portfolios obtained every month when stocks are double sorted. First, stocks are sorted according to the investment related variable enounced in the first column and quintiles are formed. Then, within each portfolio stocks are sorted according to their idiosyncratic risk level and quintiles are formed again. Panel A reports raw average monthly returns for each portfolio. Panel B reports the intercept of [10]. Both raw returns and alphas are reported in monthly percentage. The column [5-1] is the difference between portfolio 5 and portfolio 1 so that each row corresponds to a higher profitability level. Data sample covers all common stocks available jointly on CRSP and Compustat from July 1982 to December 2009. Newey-West *t*-statistics are reported in brackets.

Panel A: Quintiles portfolio returns by investment levels

Asset growth quintiles	Ranking on idiosyncratic volatility: Low 1	2	3	4	High 5	5-1
Low 1	1.36	1.26	1.57	0.56	0.48	-0.88 [-1.49]
2	1.15	1.36	1.27	1.33	1.27	0.12 [0.26]
3	1.29	1.22	1.28	1.26	1.01	-0.28 [-0.69]
4	1.39	1.40	1.29	1.11	0.71	-0.68 [-1.60]
High 5	1.12	1.16	0.80	0.11	0.08	-1.03 [-1.65]

Panel B: Quintiles portfolio alphas by investment levels

Asset growth quintiles	Ranking on idiosyncratic volatility: Low 1	2	3	4	High 5	5-1
Low 1	0.29	0.03	0.28	-0.84	-0.89	-1.18 [-2.56]
2	0.18	0.22	0.00	0.02	-0.17	-0.36 [-1.17]
3	0.37	0.10	0.10	0.15	-0.37	-0.74 [-2.59]
4	0.45	0.40	0.14	-0.06	-0.52	-0.97 [-2.74]
High 5	0.11	0.12	-0.34	-1.13	-1.45	-1.56 [-3.47]

control for the profitability effect in Table 3. Table 2 shows that although in raw returns the anomaly is only marginally significant for the last investment quintile, in risk-adjusted returns the anomaly is observed for 4 out of 5 investment quintiles. Only for quintile 2 risk adjusted returns and alpha are non-significant and equal to -0.36%. In all the other quintiles the anomaly is strong with an associated t-statistic higher than -2.00

in risk-adjusted returns. Therefore, controlling solely for the investment effect does not account for the anomaly. This evidence is in line with studies such as Jiang *et al.,* (2009) pointing accruals anomaly does not account for the idiosyncratic risk one.

Table 3: Raw and risk-adjusted returns by levels of profitability

This table reports both raw and risk adjusted returns for 25 quintile portfolios obtained every month when stocks are double sorted. First, stocks are sorted according to the profitability related variable enounced in the first column and quintiles are formed. Then, within each portfolio stocks are sorted according to their idiosyncratic risk level and quintiles are formed again. Panel A reports raw average monthly returns for each portfolio. Panel B reports the intercept of [10]. Both raw returns and alphas are reported in monthly percentage. The column [5-1] is the difference between portfolio 5 and portfolio 1 so that each row corresponds to a higher profitability level. Data sample covers all common stocks available jointly on CRSP and Compustat from July 1982 to December 2009. Newey-West *t*-statistics are reported in brackets.

Panel A: Quintiles portfolio returns by profitability levels

		Ranking on idiosyncratic volatility					
		Low 1	**2**	**3**	**4**	**High 5**	**5-1**
Tobin's Q quintiles	**Low 1**	1.26	1.76	1.86	1.92	1.37	0.11 [0.19]
	2	1.21	1.20	1.42	1.73	1.30	0.09 [-0.15]
	3	1.37	1.23	1.53	1.35	0.63	-0.75 [-1.61]
	4	1.19	1.32	1.13	0.57	0.38	-0.80 [-1.67]
	High 5	1.19	1.13	0.93	0.45	-0.11	-1.30 [-2.65]

Panel B: Quintiles portfolio alphas by profitability levels

		Ranking on idiosyncratic volatility					
		Low 1	**2**	**3**	**4**	**High 5**	**5-1**
Tobin's Q quintiles	**Low 1**	-0.10	0.23	0.31	0.48	-0.35	-0.25 [-0.59]
	2	-0.07	-0.23	-0.10	0.21	-0.21	-0.14 [-0.44]
	3	0.37	0.01	0.00	-0.27	-0.87	-1.24 [-3.92]
	4	0.25	0.33	0.01	-0.58	-0.77	-1.02 [-2.61]
	High 5	0.37	0.24	-0.01	-0.64	-1.35	-1.72 [-4.97]

Regarding profitability, Table 3 demonstrates that the anomaly is concentrated in the stocks having higher profitability; the [5-1] difference in risk-adjusted returns is -1.24% (t = -3.92), -1.02% (t = -2.61) and -1.72% (t = -4.97) for profitability quintiles 3, 4 and 5 respectively. Furthermore raw returns decrease linearly with profitability from a non-significant 0.11% in the first profitability quintile to a significant -1.30% in the last quintile. Here again, controlling for profitability does not fully account for the anomaly. The fact that the idiosyncratic risk anomaly concentrates on the higher levels of profitability is interesting given that Tobin's q provides a natural threshold (this is Tobin's q = 1) to divide the sample in two meaningful subsamples.[24] Thus, in Table 4 we create both subsamples and follow our previous methodology constructing value-weighted quintiles returns sorted by idiosyncratic volatility.

Table 4: Portfolios sorted by idiosyncratic volatility and Tobin's q values

This table reports both monthly average raw returns and monthly average risk adjusted returns for each quintile portfolio formed sorting on idiosyncratic risk using all non-financial (SIC codes 6000 – 6999) common stocks available jointly on CRSP and Compustat from July 1982 to December 2009 resulting in approximately 3.415 firms per month firms per month. Panel A includes all stocks for which Tobin's q value exceeds one, approximately 2.740 per month. Panel B include all the others, around 676 firms per month. In both panels quintile 1 corresponds to the portfolio with the lowest idiosyncratic risk and quintile 5 to the highest idiosyncratic risk. Returns and Alphas are reported in monthly percentage. The row [5-1] is the difference between portfolio 5 and portfolio 1. Newey-West *t*-statistics are reported in brackets.

	Panel A: Idiosyncratic risk returns relationship for firms having q>1				Panel B: Idiosyncratic risk returns relationship for firms having q<1		
Idiosyncratic Risk Quintiles		Returns	Alphas	Asset Growth	Returns	Alphas	Asset Growth
	Low 1	1.14	0.30 [4.05]	0.56	1.05	-0.45 [-2.14]	0.23
	2	1.19	0.14 [1.55]	0.44	1.73	0.17 [0.81]	0.96
	3	1.23	0.07 [0.61]	0.87	1.77	0.25 [1.16]	1.48
	4	0.93	-0.24 [-1.40]	2.54	1.76	0.33 [0.96]	4.05
	High 5	0.27	-1.04 [-4.05]	6.27	1.70	-0.09 [-0.23]	1.84
	5-1	-0.96 [-2.16]	-1.34 [-4.39]	5.72 [2.20]	0.65 [1.11]	0.36 [0.86]	1.61 [1.39]

[24] Tobin's q provides a measure of market valuation of a company. A company having a Tobin's q larger than one should be perceived as a more profitable one in the sense that the firm has incentives to invest in future projects.

Both Panel A and Panel B of Table 4 report returns, alphas and asset growth in percentage terms. Panel A corresponds to firms with Tobin's q larger than one while Panel B to firms with Tobin's q lower than one. As expected from our previous results,the [5-1] difference is significant and negative for the subsample of firms with higher profitability (Tobin's q larger than one) while is positive and non-significant for the other one. In terms of raw returns (alphas), the difference across extreme quintiles is equal to -0.96% (-1.34%) with a significant t-statistic of -2.16 (-4.39) for the subsample with higher profitability and equal to an insignificant 0.65% (0.36%) for firms with lower profitability level. Remarkably, the [5-1] difference in asset growth is significant only in the subsample where the idiosyncratic volatility anomaly is observed. This fact highlights the importance of taking into account the profitability effect when attempting to link the idiosyncratic volatility anomaly to corporate investment in the sense that it might be the case that investment fully accounts for the idiosyncratic volatility anomaly once the profitability effect is discounted and that this link is shadowed when controlling only for investment. In the following section, we test our hypothesis by introducing simultaneous controls for profitability and investment in the cross-section analysis.

3.4. The idiosyncratic risk – investment relationship conditional to profitability

We investigate the relation between idiosyncratic volatility and expected returns after jointly control for the investment and profitability effects by examining the sign and statistical significance of the mean value of γ_1, the coefficient on the idiosyncratic volatility measure in:

$$\begin{aligned} r_{it} = \alpha_{it} + \gamma_1 \sigma_{it-1} + \gamma_2 \beta_{1it} + \gamma_3 \beta_{2it} + \gamma_4 \beta_{3it} + \gamma_5 s_{it-6} + \gamma_6 lr_{it-6} \\ + \gamma_7 q_{it-6} + \gamma_6 ag_{it-6} + \xi_{it}. \end{aligned} \qquad [11]$$

where current month stock excess returns (r_t) are regressed onto one-month previous idiosyncratic risk (ε_{t-1}), current loadings on the market factor (β_1), the SMB factor (β_2) and the HML factor (β_3) of a Fama and French (1993) model, size (s), lag returns over the previous six months (lr) and both our accounting variables, Tobin's q (q) and asset growth (ag) which are built with data available to the market during the last 6 months.

We estimate alternative models changing the definition of profitability to “Tobin’s q var”, to “Tobin’s q min”, to profitability, to dividends or to a dummy variable equal to one for dividend paying firms. All these variables were defined over the previous section.[25]

Table 5 reports results of the Fama and MacBeth (1973) regressions in equation [11]. The first column, named Model (1) should present the puzzle before controlling by corporate variables. Model (2) corresponds to the hypothesis test of the investment effect on the idiosyncratic volatility anomaly from the point of view of the accruals anomaly. Although our investment measure is different from the one used by Jiang *et al.,* (2009) this model should be unable to account for the significance of the idiosyncratic risk since no controls for profitability are included. Models (3) to (8) present the jointly effect over γ_1 of controlling by investment and profitability using the different proxies we propose for profitability. Although previous literature includes a five lags Newey-West adjustment to account for heteroscedasticity all our regressions include a two lags adjustment since the five lags usual adjustment largely reduces the anomaly and therefore plays in our favor.

Thus, Model (1) only indicates that consistently with Table 1 the anomaly is observed in our data; the coefficient related to idiosyncratic volatility is equal to -0.0750 and statistically significant. On the other hand, the results of Model (2) are consistent with our hypothesis since they show that controlling only for investment γ_1 is still significant, and therefore the idea that investor mispricing related to overreaction to past information such as change in total assets or accruals is not the underlying force driving the anomaly. Once this possibility is discarded from our sample, Models (3) to (8) that correspond to our hypothesis merit some discussion. The basic result is provided by Model (3) that shows that controlling for the joint effects of profitability and investment accounts for the idiosyncratic volatility anomaly so that γ_1 becomes non-significant with a t-statistic equal to -1.63. Furthermore, both Tobin’s q and asset growth measures proof to be largely significant with t-statistic equal to -4.53 for the former and -2.95 for the latter. Although the sign attached to the coefficient related to Tobin’s q is contrarian to the expected sign we argue that this sign can become negative after additional

[25] There is evidence showing that the anomaly is robust to the estimation of idiosyncratic risk using a Carhart (1997) model. For further details see the first note foot on this paper.

Table 5: Fama-MacBeth Regressions Controlling Simultaneously for Profitability and Investment Effects

This table reports the results of Fama-MacBeth regressions using all non-financial (SIC codes 6000 – 6999) common stocks available jointly on CRSP and Compustat from July 1982 to December 2009. The dependent variable is monthly stock returns compounded from daily data. The explanatory variables are idiosyncratic volatility (ε), $\hat{\beta}_{mkt}$, $\hat{\beta}_{smb}$, $\hat{\beta}_{hml}$, size, lag returns over the previous six months, Asset growth, Tobin's q, Tobin's q var, Min Tobin's q corresponding to the minimum value between Tobin's q and Tobin's q var, dividends per share and a dummy variable equal to one for dividend paying firm. t-statistics are reported in parentheses.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
σ_{t-1}	-0.0750**	-0.0705*	-0.0584	-0.0675*	-0.0527	-0.0696*	-0.0633*	-0.0548
	(-2.006)	(-1.898)	(-1.633)	(-1.820)	(-1.472)	(-1.878)	(-1.751)	(-1.571)
	0.00341***	0.00340***	0.00354***	0.00340***	0.00356***	0.00339***	0.00362***	0.00369***
	(3.456)	(3.474)	(3.670)	(3.469)	(3.703)	(3.459)	(3.740)	(3.880)
	0.00145***	0.00148***	0.00152***	0.00149***	0.00154***	0.00149***	0.00150***	0.00150***
	(2.938)	(2.987)	(3.093)	(3.005)	(3.128)	(3.004)	(3.048)	(3.063)
	-0.00105*	-0.00106*	-0.00118**	-0.00108*	-0.00122**	-0.00107*	-0.00117**	-0.00121**
	(-1.815)	(-1.839)	(-2.108)	(-1.874)	(-2.167)	(-1.842)	(-2.051)	(-2.166)
Size	-0.00213***	-0.00212***	-0.00194***	-0.00211***	-0.00193***	-0.00213***	-0.00274***	-0.00269***
	(-4.135)	(-4.136)	(-3.707)	(-4.124)	(-3.715)	(-4.148)	(-5.268)	(-5.481)
Lag Returns	0.00189	0.00184	0.00130	0.00196	0.00165	0.00182	0.00207	0.00200
	(0.755)	(0.738)	(0.524)	(0.807)	(0.679)	(0.728)	(0.829)	(0.803)
Asset Growth		-0.000387***	-0.000327***	-0.000387***	-0.000308***	-0.000389***	-0.000348***	-0.000326***
		(-3.263)	(-2.947)	(-3.272)	(-2.763)	(-3.276)	(-3.066)	(-2.903)
Tobin's q			-0.00123***					
			(-4.533)					
Tobin's q var				-0.000179**				
				(-2.048)				
Min Tobin's q					-0.00286***			
					(-7.377)			
Profitability						5.16e-05		
						(0.592)		
Dividends							0.00443***	
							(4.395)	
DumDiv								0.00526***
								(3.502)
Constant	0.0157***	0.0157***	0.0166***	0.0159***	0.0182***	0.0157***	0.0167***	0.0155***
	(3.515)	(3.530)	(3.748)	(3.563)	(4.073)	(3.539)	(3.817)	(3.504)
Observations	1,004,965	1,004,965	1,004,965	1,004,965	1,004,965	1,004,965	1,004,965	1,004,965

t-statistics in parentheses
*** p<0.01, ** p<0.05, * p<0.1

controls such as the estimated betas from the asset pricing model or size are included in the regression.

Results for the alternative measures of profitability, Models (4) to (8), are mixed. In Models (3), (5) and (8) profitability and investment controls are successful in fully accounting for the anomaly while in Models (4), (6) and (7) these controls are only able to diminish the anomaly to a 10% significance. We hypothesize that the relative failure of the latter models lies in the fact that profitability is a broad concept and therefore difficult to measure or observe directly. In this context, different proxies might be capturing different features of profitability or might be better than others. However this disappointment is mitigated by unreported results showing that including two measures of profitability together (i.e. adding either dividens or the dummy variable for dividend paying firms into these models), or that increasing the number of lags for the Newey-West adjustment fully accounts for the anomaly. These results are available upon request. Altogether, we believe the evidence strongly supports our hypothesis that joint controls for profitability and investment proof able to account for the idiosyncratic risk anomaly.

To provide further insights we profit from the fact that, as discussed earlier, the hypothesis that investment and profitability can jointly account for the anomaly should hold both with rational or irrational expectations so that no mispricing is necessary. We therefore explore how our results behave in times of high investor sentiment when irrational expectations are more likely to be latent and, in times of low investor sentiment when rational expectations should prevail. We include two additional multiplicative effects in [11]: (i) $\sigma_{t.1}$_HighSent, measuring the interaction between a dummy variable equal to one if the previous month is considered a high sentiment month and the idiosyncratic volatility in the previous month and (ii) $\sigma_{t.1}$_LowSent, the interaction between a dummy variable equal to one if the previous month is considered a low sentiment month and the idiosyncratic volatility in the previous month. A month is considered a high (low) sentiment period if the value of the Baker and Wurgler Index is positive (negative).[26] The resulting model can be written as:

$$r_{it} = \gamma_1 \sigma_{it-1} DHighS + (1 - DHighS)\gamma_2 \sigma_{it-1} + \gamma_3 \beta_{1it} + \gamma_4 \beta_{2it} + \gamma_5 \beta_{3it} + \gamma_6 s_{it-6} + \gamma_7 lr_{it-6} + \gamma_8 DHighS + \gamma_9 (1 - DHighS) + \gamma_{10} q_{it-6} + \gamma_{11} ag_{it-6} + \zeta_{it} \quad [12]$$

where, $DHighS$ is a dummy variable equal to one if month t-1 is considered to be a high investor sentiment month and the rest of the variables are defined as in equation [11].

[26] For details on the construction of the Index see Baker and Wurgler (2006 and 2007). We also perform the analysis defining a high (low) sentiment period any month in which the value of the index is higher (lower) than one standard deviation from the mean of the index. Since results do not change substantially, we only tabulate the former in order to make our results comparable to Gao *et al*., (2012).

Table 6: Fama-MacBeth Regressions: Investor Sentiment

This table reports the results of Fama and MacBeth regressions using all non-financial (sic codes 6000 – 6999) common stocks available jointly on CRSP and Compustat from July 1982 to December 2009. The dependent variable is monthly stock returns compounded from daily data. The explanatory variables are idiosyncratic volatility (ε), $\hat{\beta}_{mkt}$, $\hat{\beta}_{smb}$, $\hat{\beta}_{hml}$, size, lag returns over the previous six months, asset growth, Tobin's q, Tobin's q var, Min Tobin's q corresponding to the minimum value between Tobin's q and Tobin's q var, dividends per share and a dummy variable equal to one for dividend paying firm. ε_{t-1}_HighSent is the interaction between a dummy (D High Sent) equal to one if the investor sentiment index by Wurgler *et al.,* (2006) is positive in the precedent month and the idiosyncratic volatility (ε), ε_{t-1}_LowSent is the interaction between a dummy (D Low Sent) equal to one if the investor sentiment index by Wurgler *et al.,* (2006) is negative in the precedent month and the idiosyncratic volatility (ε). t-statistics are reported in parentheses.

	(8)	(9)	(10)	(11)	(12)	(13)	(14)
σ_{t-1}_HighSent	-0.0272**	-0.00271	-0.0144	-0.00938*	-0.00896	-0.0268***	-0.00685
	(-2.465)	(-0.211)	(-1.484)	(-1.843)	(-1.283)	(-3.025)	(-1.233)
σ_{t-1}_LowSent	-0.0479	-0.0557*	-0.0531	-0.0433	-0.0606*	-0.0365	-0.0480
	(-1.357)	(-1.663)	(-1.479)	(-1.257)	(-1.655)	(-1.071)	(-1.397)
	0.00341***	0.00354***	0.00340***	0.00356***	0.00339***	0.00362***	0.00369***
	(3.456)	(3.670)	(3.469)	(3.703)	(3.459)	(3.740)	(3.880)
	0.00145***	0.00152***	0.00149***	0.00154***	0.00149***	0.00150***	0.00150***
	(2.938)	(3.093)	(3.005)	(3.128)	(3.004)	(3.048)	(3.063)
	-0.00105*	-0.00118**	-0.00108*	-0.00122**	-0.00107*	-0.00117**	-0.00121**
	(-1.815)	(-2.108)	(-1.874)	(-2.167)	(-1.842)	(-2.051)	(-2.166)
Size	-0.00213***	-0.00194***	-0.00211***	-0.00193***	-0.00213***	-0.00274***	-0.00269***
	(-4.135)	(-3.707)	(-4.124)	(-3.715)	(-4.148)	(-5.268)	(-5.481)
Lag Returns	0.00189	0.00130	0.00196	0.00165	0.00182	0.00207	0.00200
	(0.755)	(0.524)	(0.807)	(0.679)	(0.728)	(0.829)	(0.803)
D High Sent	0.00799**	0.00854**	0.00808**	0.00957***	0.00804**	0.00863***	0.00778**
	(2.377)	(2.547)	(2.415)	(2.811)	(2.409)	(2.609)	(2.333)
D Low Sent	0.00769**	0.00809***	0.00785**	0.00861***	0.00766**	0.00806***	0.00771**
	(2.535)	(2.674)	(2.560)	(2.824)	(2.534)	(2.693)	(2.567)
Asset Growth		-0.000327***	-0.000387***	-0.000308***	-0.000389***	-0.000348***	-0.000326***
		(-2.947)	(-3.272)	(-2.763)	(-3.276)	(-3.066)	(-2.903)
Tobin's q		-0.00123***					
		(-4.533)					
Tobin's q var			-0.000179**				
			(-2.048)				
Min Tobin's q				-0.00286***			
				(-7.377)			
Profitability					5.16e-05		
					(0.592)		
Dividends						0.00443***	
						(4.395)	
DumDiv							0.00526***
							(3.502)
Observations	1,004,965	1,004,965	1,004,965	1,004,965	1,004,965	1,004,965	1,004,965

t-statistics in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Here again, we estimate alternative models changing the definition of profitability to "Tobin's q var", to "Tobin's q min", to profitability, to dividends or to a dummy variable equal to one for dividend paying firms. Notice also that to avoid collinearity problems the model in equation 12 does not include a constant. The results of the Fama-Macbeth estimations are presented in Table 6 and also support our hypothesis. Model in equation [11] separates the relationship between expected returns and idiosyncratic volatility in two so that it becomes possible to test the hypothesis that investment and profitability controls are able to account for the anomaly also during times of high investor sentiment. The relationship achieved just after periods of high investor sentiment is isolated by $\gamma_1$and the one observed just after periods of low investor sentiment is measured by γ_2. Consistently with the findings of Gao *et al.,* (2012) Model (8) shows that the relationship is only significant for the times after sentiment has been high. For these periods, $\gamma_1$is equal to -0.0272 and with a t-statistic of -2.47 it is largely significant. For the remaining periods, the idiosyncratic risk – expected returns relationship is also negative (-4.79%) but is not statistically significant. Our results, however, show that the anomaly does not particularly depend on investor mispricing during times of high sentiment. Actually, Model (9) shows that controlling for investment and profitability totally account for the anomaly during these periods while it makes the anomaly marginally significant for periods following low investor sentiment. This results seem quite robust since in 4 out of the 6 models considered the anomaly totally disappears for times succeeding high investor sentiment while it remains unobserved or is only marginally significant (p-values close to 0.10) for times succeeding low investor sentiment.

3.5. Conclusions

A considerable body of the literature related to the idiosyncratic volatility anomaly suggests it is related to several motifs such as investor preferences for particular types of stocks or to investor mispricing. Surprisingly, the role of corporate in the anomaly has been largely neglected. Approaching the anomaly from corporate investment is particularly appealing because it is plausible to argue that investment is the result of a totally idiosyncratic decision making process and large investments should increase uncertainty on the firm performance. Moreover, valuation theory offers a testable theoretical framework in which investment and expected returns should be negatively related once profitability is accounted for.

In this paper we test the hypothesis that the idiosyncratic volatility anomaly might be observed because the measure of idiosyncratic risk could be contaminated by the effects of investment and profitability. Our results show that both corporate investment and profitability are significantly related to returns and that the idiosyncratic volatility anomaly is no longer observed after these effects are accounted for. In particular our results contradict the idea that the idiosyncratic risk anomaly is related to irrational investor expectations led either by a misunderstanding of the information content of cash-flows or by euphoria during times of high sentiment. More interesting is the fact that we follow a theoretical approach that permit us to test several hypotheses previously considered in the literature and to show why, although our results do not contradict the empirical results found in these previous studies, their interpretations are not sound. We believe our interpretation is somehow broader in its implications.

Chapter IV: Idiosyncratic volatility, conditional liquidity and stock returns

4.1. Introduction

As counterintuitive as it might appear recent literature shows that unsystematic risk is relevant and seems to have increase over time. In the last decade a number of papers have discussed whether the apparent upward trend in idiosyncratic volatility reported by Campbell *et al.*, (2001) is real or, just an illusion driven by the lack of controls related to returns such as available growth options (Cao *et al.*, 2008) or firms' profitability (Pastor and Varonesi, 2003, Wei and Zhang, 2006). Independently of its evolution over time there is no doubt that idiosyncratic risk, defined as the standard deviation of the residuals of a CAPM or a Fama and French (1993) model, has become a relevant topic given that its information content is larger than theoretically anticipated. In this sense, idiosyncratic risk has been shown to increase with expected earnings growth (Campbell *et al.*, 2001 and Xu and Malkiel, 2003), to be related with business cycles in a countercyclical way (Brown and Ferreira, 2004) and to correlate negatively with liquidity (Spiegel and Wang, 2005). All this can be seen as a critique to the CAPM and to the Fama and French (1993) model as accurate asset pricing models. In particular, it appears that their specifications miss an element that is thus captured in the residuals making them relevant against theoretical arguments.

Out of all the discussions related to the idiosyncratic volatility, the idiosyncratic volatility anomaly is one of the stronger critique to the CAPM type models and also the most controversial one. The anomaly, or what is the same, the fact that unsystematic risk is negatively correlated with subsequent returns (Ang *et al.*, 2006 and 2009), implies a negative relationship between risk and returns that is difficult to explain in the CAPM's framework and that constitutes a dynamic field of research within which this paper is situated. Despite the controversy it initially generated, the few papers arguing the anomaly is not robust (Bali and Cakici, 2008) or mistakenly conceived (Fu, 2009) have been surpassed by a large number of papers trying to explain this empirical observation. In fact, the anomaly is now well established and potential explanations for it lie in papers arguing it is related to non-synchronicity of trading (Han and Lesmond, 2011), to investor sentiment (Gao *et al.*, 2012) or to the preference of investors towards stocks offering features such as positive skewness (Kapadia, 2006 and Boyer *et al.*, 2010) or lottery-like payoffs (Bali *et al.*, 2011) among others.

Our approach is based on two key points in the discussion on the idiosyncratic risk anomaly; on the one hand liquidity and, on the other, the fact that the anomaly is not observed at all periods of time. The presence of liquidity as a potential explanation for the anomaly has been a noticeable element in the analysis of the relationship between idiosyncratic risk and expected returns from the seminal paper by Ang et al. (2006) that were the first to rule-out it as the factor driving the anomaly. To do so, authors included ex-post controls for liquidity and conclude the idiosyncratic volatility anomaly is observed for all quintiles of liquidity.[27] However, more recently Han and Lesmond (2011) and Han *et al.,* (2011) argue that illiquidity has a major impact in the estimation of the idiosyncratic risk itself so that the illiquidity effect has to be accounted for before estimating the actual measure of idiosyncratic risk.

On the other hand, the fact that the anomaly is not pervasive at all times was recently evidenced by Gao *et al.*, (2012) who dividing their sample in periods following times of high and low investor sentiment show the anomaly is significant only after periods of high investor sentiment. In this paper we also use the idea that the anomaly is not pervasive over time and we show it is indeed observed after good economic times but disappears after distress times. At first sight this fact could seem to be aligned with the argument by Gao *et al.*, (2012) that the anomaly is the manifestation of a mispricing effect lead by sentiment investors vis-à-vis arbitrageurs. It would seem natural that the anomaly is only observed after good economic times because in any other time arbitrageurs would overrule sentiment

[27] These controls are ex-post in the sense that the estimation of idiosyncratic risk does not depend on the illiquidity of the stock. Controls for illiquidity are included only after the quintiles based on idiosyncratic risk are built and intend to test if the spread in returns of portfolios with different levels of idiosyncratic risk can be related to a spread in the illiquidity of these portfolios.

investors and drive prices to reflect fundamental values. However, in a recent paper Malagón *et al.*, (2013b) provide an alternative theoretical approach to the negative relationship between idiosyncratic risk and expected returns that does not imply mispricing and show that simultaneous controls for corporate profitability and investment account for the anomaly also during periods of high investor sentiment. If sentiment is dropped from the equation it becomes difficult to explain why the anomaly is conditional on the state of the economy in such a way that it is only relevant for periods following normal times.

In this paper we conjecture that the explanation might settle on the conditional pricing of liquidity shocks during times of financial distress recently evidenced by Acharya *et al.*, (2012). Our rational is as follows. Since according to Spiegel and Wang (2005) idiosyncratic risk and liquidity are correlated negatively then it should be that the portfolio with the highest level of idiosyncratic risk is more illiquid than the one with the lowest level. The flight to liquidity phenomenon could then apply to the extreme idiosyncratic volatility portfolios.[28] If this is so, during recessions the stocks forming the portfolio with the highest level of idiosyncratic risk (illiquid stocks) should tend to depreciate while the ones on the portfolio with the lowest level of idiosyncratic risk (liquid stocks) should tend to appreciate. In the following periods both groups of stocks should suffer a correction in process as the economic regime changes or the liquidity shock disappears so that the returns of the high idiosyncratic risk portfolios should increase and the ones of the low idiosyncratic risk should decrease. In this framework, the differences of returns between the high and the low idiosyncratic risk portfolios could become not significant and even positive as it is indeed observed.

In order to test our hypothesis we estimate a Markov regime switching model allowing for different return structures for high and low idiosyncratic risk portfolios and for different loadings on the variables used to define these return structures over two distinct economic

[28] Flight to liquidity occurs during recessions because investors risk aversion increases making them changing their holdings of illiquid securities to more liquids ones. Literature has checked this issue showing that investors change the securities not only from stocks to bonds, but also within stocks. During crisis mutual funds also reduce their holdings of illiquid assets and Hedge Funds trading patterns change (see for example Longstaff (2004), Ben-Repahel (2011) or Ben-David *et al.,* (2010)).

regimes (normal times and recessions). Overall, our results support the idea that liquidity shocks affect stocks with high and low idiosyncratic risk in opposite ways during recessions. Moreover, the effect of liquidity shocks is such that there is a flight to liquidity and that the returns of the quintile with the lowest level of idiosyncratic risk increase significantly during recessions. This result supports our hypothesis that there is a flight to liquidity that results in the idiosyncratic volatility anomaly disappearing in periods following recessions as the returns of the portfolio with the lowest level of idiosyncratic risk suffer a correction in prices once the economy is no longer under stress.

The remainder of the paper is organized as follows. In Section 2 we discuss the relevance of liquidity in the discussion related to the idiosyncratic volatility anomaly and link it to economic regimes. Then, in Section 3 we use several measures of economic conditions and several sample periods to demonstrate that the anomaly is not pervasive over time, being significant only in periods following normal economic times. Section 4 shows the results obtained estimating a Markov regime-switching model to check the conditional effect of liquidity shocks both for high and low idiosyncratic volatility stocks in alternative economic regimes. Finally, Section 5 concludes.

4.2. Idiosyncratic risk anomaly, liquidity and economic times

The most common way to show the idiosyncratic volatility anomaly is to rank stocks according to their level of idiosyncratic risk and to form quintiles portfolios so that the fifth quintile contains the highest idiosyncratic risk stocks. Once this is done, the anomaly is observed when comparing the value weighted returns of the extreme quintile portfolios since the [5-1] difference in returns is negative and significant. The fact that the idiosyncratic risk anomaly is observed can either imply that there is a missing factor in the asset pricing model or, alternatively, that there is a characteristic shared by the stocks with higher idiosyncratic risk able to explain the spread in returns of the extreme idiosyncratic risk quintiles of stocks. In this context one of the variables that could potentially explain the idiosyncratic volatility anomaly is liquidity since it has been shown that both aggregate liquidity and the liquidity level of each stock are related to returns. The relevance of liquidity as a pricing factor was shown by Pastor and Stambaugh (2003) who provide evidence that aggregate liquidity is a priced factor in the stock market and that stocks with low liquidity betas have relatively lower returns. Also, several papers such as Amihud and Meldenson (1986), Brennan and Subrahmanyam (1996), Brennan *et al.*, (1998), Datar *et al.*, (1998) and Fiori (2000) show that, in average, illiquid stocks tend to have higher returns since investors have a preference for liquidity. Therefore, both options considered above are plausible and liquidity could account for the anomaly in several ways. First, if the anomaly is related with liquidity as a pervasive pricing factor then the stocks with higher idiosyncratic risk should have low liquidity betas. Second, if stocks with high idiosyncratic risk are particularly illiquid then the level of liquidity of these stocks could be a characteristic explaining the anomaly.

The relevance of liquidity as a potential explanation for the anomaly has been indeed latent from the seminal paper by Ang *et al.,* (2006) who control for liquidity performing double sorts on characteristics related to it and on idiosyncratic risk. Their controls for liquidity include liquidity betas based on Pastor and Stambaugh (2003), volume, turnover measured as volume divided by total shares outstanding and the bid-ask spread. In all cases the particular liquidity control is unable to account for the anomaly so that authors conclude their results are robust to liquidity. A similar conclusion is found by Spiegel and Wang (2005) who using several proxies for liquidity show that liquidity and idiosyncratic risk are negatively correlated and that, although both have effects on the cross-section of stock returns, the effect of idiosyncratic risk dominates the one of liquidity for all proxies. However, two recent papers argue the controls for liquidity have to be included before estimating the idiosyncratic volatility and show that liquidity is in the core of the anomaly. Han and Lesmond (2011) argue liquidity affects the estimation of idiosyncratic volatility via the percentage of zero returns which affects the loadings on the systematic risk factors, and via the bid-ask spread that increases the variance of the returns. Therefore, authors perform double sorts first on the percentage of zero returns during a month and then on idiosyncratic risk and estimate the idiosyncratic volatility using midpoint returns to control for the bid-ask bounce. They conclude that both approaches are able to fully account for the significance of the explanatory power of idiosyncratic risk on returns and argue their results highlight the relevance of liquidity in the discussion. [29] Their results are reinforced by Han *et al.,* (2011) who show the midpoint approach accounts for the anomaly in 45 markets including 22 emerging ones. It seems then that although ex-post controls for liquidity are not effective to account for the idiosyncratic risk anomaly, the effect of liquidity on the estimation of the idiosyncratic risk should be taken into account when studying it.

A noticeable fact that has not been covered in the literature relating liquidity with the idiosyncratic risk anomaly is the fact that the anomaly is not observed at all times. Authors such as Gao *et al.,* (2012) show that the anomaly is only observed after periods of high investor sentiment and conclude the anomaly arises from irrational investors who are able to overrule arbitrageurs only during times of high sentiment. In the next section we go further and using several proxies for the state of the economy we demonstrate that, in general, the anomaly vanishes after recession times. The fact that the anomaly is not pervasive over economic regimes is difficult to justify using the sentiment approach followed by Gao *et al.,* (2012) because normal times do not necessarily imply a stronger influence of sentiment investors vis-à-vis arbitrageurs and given the recent evidence against the sentiment hypothesis (Malagon *et al.,* 2013b). It is also difficult to find a theoretical argument based on the microstructure arguments provided by Han and Lesmond (2011) or Han *et al.,* (2011) fitting this observation since the liquidity related problems they consider should increase during recessions when liquidity tends to be scarce so that if the anomaly was to be explained through liquidity it should become stronger during these times. A possible answer to this issue might be found in a recent paper by Acharya *et al.,* (2012) who demonstrate the effect of liquidity on stock returns is conditional on the state of the economy, that liquidity shocks affect asset prices in a stronger way during recessions and that during these times there is a flight to liquidity throughout which the prices of liquid assets tend to raise and the prices of illiquid assets tend to decline. In this context, the estimation of the idiosyncratic risk might also be sensitive to the economic conditions characterizing the particular time over which this risk is estimated. In particular, and given the fact that Spiegel and Wang (2005) showed that stocks with higher (lower) idiosyncratic

[29] A recent paper by Chen *et al.,* (2012a) argues the percentage of zero returns does not account for the anomaly. However, since it does not provide any argument related to liquidity we do not refer to it extensively here.

risk are in general less (more) liquid, it is important to analyze whether the flight to liquidity phenomenon documented by Acharya *et al.,* (2012) has an effect on the anomaly.

Our hypothesis in this paper is that during recessions liquidity shocks affect the returns of extreme idiosyncratic risk portfolios in opposite directions shrinking their difference. In particular, we hypothesize that during bad economic times both the portfolios with the lowest and the highest levels of idiosyncratic risk are subject to a flight to liquidity phenomenon that translates in the prices of stocks with highest (lowest) idiosyncratic risk tending to depreciate (appreciate). If this is true, stocks forming these portfolios should suffer a correction in prices once the economic regime changes increasing the returns of the riskier portfolio (in terms of idiosyncratic risk) and decreasing the ones of the less risky portfolio. Then, following recessions the spread in returns of the portfolio with the highest level and the one with the lowest level of idiosyncratic risk should diminish and could either become non-significant or even positive.

4.3. Preliminary evidence

In this section we confirm the idiosyncratic volatility – expected returns puzzle is observed in our sample. Each month, we sort stocks according to their idiosyncratic volatility estimated over the previous six months, defined as the standard deviation of the residuals, ($\sigma_{\varepsilon_t^i}$), in the three-factor model of Fama and French (1993):

$$r_t^i = \alpha^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \varepsilon_t^i , \qquad [1]$$

where, r_t^i is the stock returns in excess of the risk free rate and $\{MKT_t, SMB_t, HML_t\}$ represent the market, size and book to market factors.[30,31] Once stocks are sorted into quintiles, where the first one contains stocks with the lowest risk and the last one those with

[30] The original methodology by Ang *et al.,* (2006) implies the estimation of the idiosyncratic risk only over one month. However, we use the estimation over six months to address the critique of error-in-variance exposed by Malkiel and Xu (2002). Using this estimation goes against our interest because the anomaly is much more significant using the six month estimation of idiosyncratic risk.
[31] The factors used the model have been obtained from Kenneth French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library html

the highest risk, quintiles portfolios are formed and hold for one month. The corresponding portfolios are value-weighted and rebalanced month by month. Our database includes daily returns of all stocks in the CRSP (*Chicago Research Stock Prices*) for NYSE, AMEX and NASDAQ markets from July 1963 to December 2009.

Table 1 reports the results we obtain using data from January 1964 to December 2009 since the first six months are lost in the initial estimation of idiosyncratic risk. The table reports monthly average returns, standard deviations, and alphas (all in percentage) for portfolios sorted on idiosyncratic volatility. Alphas CAPM correspond to Jensen's alphas calculated with respect to the CAPM and Alphas FF with respect to the three-factor model. The t-statistics are reported in brackets. The row [5-1] is the difference between portfolio 5 and portfolio 1 where Newey-West t-statistic is also reported in brackets.

Table 1: Returns of portfolios sorted by idiosyncratic risk

This table reports the results we obtain by forming quintile portfolios according to idiosyncratic risk, estimated over 6 months, using data from July 1963 to December 2009. Since the six initial months are lost, effectively the table corresponds to data from January 1964. Quintile 1 corresponds to the lowest idiosyncratic risk portfolio and quintile 5 to the highest idiosyncratic risk. Returns and standard deviation (Std Dev) are reported in monthly percentage. The row [5-1] is the difference between portfolio 5 and portfolio. Alphas CAPM correspond to Jensen's alphas calculated with respect to the CAPM and Alphas FF with respect to the three-factor model and are also reported in percentage. Newey-West t-statistics are reported in brackets. * denotes significance at 10% level, ** significance at 5% level and *** significance at 1% level.

Quintile	Returns	Std Dev	Alphas CAPM	Alphas FF
1	0.91	3.88	0.12 [2.26]	0.11 [2.74]
2	0.99	5.30	0.06 [1.17]	0.02 [0.36]
3	1.05	6.88	0.00 [0.14]	0.03 [0.42]
4	0.66	8.61	-0.04 [-2.43]	-0.59 [-3.81]
5	0.12	9.83	-1.01 [-3.66]	-1.17 [-5.84]
[5-1]	-0.79** [-2.09]		-1.59*** [-4.98]	-1.73*** [-7.76]

Once stocks are sorted on idiosyncratic volatility average returns of quintile portfolios display an inverse U-shaped form increasing in the middle quintiles; returns rise from

0.91% in quintile 1 to 1.05% in quintile 3, then drop to 0.12% in quintile 5. The difference [5-1] is in average -0.79% per month and statistically significant. Moreover, Jensen's alphas are positive for the initial three portfolios and switch to negative from the fourth. Both [5-1] differences in CAPM alphas and in FF alphas are negative, -1.59% the former and -1.73% the latter, showing the puzzle appears even after controlling for risk. These results exhibit similar patterns to the ones reported by Ang *et al.,* (2006) and provide evidence that the idiosyncratic volatility anomaly is robust since it is observed for a longer period and is not modified by the particularly unstable times characterizing the late years of our sample or by the use of a longer period in the estimation of the idiosyncratic risk.

Since the anomaly is observed in our sample we can proceed to show that it is not pervasive over time and that it depends on the economic regime characterizing a particular moment. To identify recessions we define three dummy variables, one based on the NBER Business Cycle data, one on the Kansas City Financial Stress Index and one on the Saint-Louis Fed Financial Stress Index. The NBER Business Cycle data directly defines expansion and recession months so the construction of the dummy is straightforward; it takes the value one when the previous month is classified as recession. Both the Kansas City Financial Stress Index and the Saint-Louis Financial Stress Index reflect the level of financial stress in the American economy so that high values of the indexes correspond to high financial stress. Intuitively, high financial stress is related to economic downturn so that the dummies related to each of these indexes take the value one when the particular index value for the previous month is larger than its historical mean. Using each of these variables we separate normal and recession months and we check whether the anomaly is observed or not for each of these economic regimes. Results, reported in Table 2, show that the anomaly is not pervasive over time disappearing just after recessions.

Panel A shows the monthly percentage returns for idiosyncratic risk quintiles both for months following recessions and for months following expansions as defined in the NBER business cycle data. The data sample covers the period between January 1964 and December 2009. The difference between the fifth and the first quintile portfolio is equal to

Table 2: Returns of portfolios sorted by idiosyncratic risk both for periods following recessions and for periods following expansions

This table reports the results we obtain by forming quintile portfolios according to idiosyncratic risk and separating the sample months into periods following recessions and periods following expansions. Quintile 1 corresponds to the lowest idiosyncratic risk portfolio and quintile 5 to the highest idiosyncratic risk. In Panel A months are classified into these two regimes according to the NBER Business Cycle Data and covers the sample period from January 1964 to December 2009. In Panel B months are classified into the two regimes according to the Kansas City Financial Stress Index so that any month having a value higher that the historical mean of the index is considered as a recession period. It covers the sample period from February 1990 to December 2009. Panel C follows the same logic but classification is done following the Saint-Louis Financial Stress Index and the sample covers the period from December 1993 to December 2009. In all tables, returns and standard deviation (Std Dev) are reported in monthly percentage. The row [5-1] is the difference between portfolio 5 and portfolio. Alphas CAPM correspond to Jensen's alphas calculated with respect to the CAPM and Alphas FF with respect to the three-factor model. Newey-West t-statistics are reported in brackets and p-values in parenthesis. * denotes significance at 10% level, ** significance at 5% level and *** significance at 1% level.

Panel A: Quintiles by Idiosyncratic Risk in Recessions and Expansions (NBER)

Idiosyncratic Risk Quintiles	after recession months: Returns	after expansion months: Returns
Small 1	0.34	1.02
2	0.18	1.14
3	0.33	1.19
4	-0.19	0.83
High 5	-0.88	0.32
5-1	-1.22 *[-1.11]*	-0.70 *[-1.79]*

Panel B: Quintiles by Idiosyncratic Risk in differing Financial stress regimes (Kansas Index)

	after recession months		after expansion months	
Idiosyncratic Risk Quintiles		Returns		Returns
	Small 1	0.60	**Small 1**	1.07
	2	0.50	**2**	1.11
	3	1.06	**3**	1.05
	4	0.71	**4**	0.54
	High 5	0.84	**High 5**	0.07
	5-1	0.24 *[0.15]*	**5-1**	-1.00 *[-1.92]*

Panel C: Quintiles by Idiosyncratic Risk in differing Financial stress regimes (St Louis Index)

	after recession months		after expansion months	
Idiosyncratic Risk Quintiles		Returns		Returns
	Small 1	0.03	**Small 1**	1.25
	2	0.02	**2**	1.10
	3	0.50	**3**	1.10
	4	0.38	**4**	0.51
	High 5	1.83	**High 5**	-0.51
	5-1	1.80 *[0.92]*	**5-1**	-1.76 *[-1.97]*

a significant -0.70% (t-stat = -1.79) for the time periods following expansions while it is equal to -1.22% but insignificant (t-stat = -1.11) for the ones following recessions. The anomaly is therefore only significant at 10% during times following expansions and the results seem to only weakly point out that the anomaly is not pervasive over time. However, using two alternative definitions of economic regime both Panels B and C provide stronger evidence on the influence recessions have on the idiosyncratic volatility anomaly. On the one hand, Panel B displays the results when normal and recession months are separated according to the Kansas City Financial Stress Index that covers the period from February 1990 to December 2009. In this panel, any month for which the index is higher than its historical mean is considered as a recession period. In this case again, the anomaly is only observed for the group of months following expansions for which the difference in returns of extreme quintiles of idiosyncratic risk is equal to -1.00% and significant at 5% with a t-stat of -1.92. Moreover, in the case of periods following recessions the [5-1] difference between extreme quintiles of idiosyncratic risk is actually positive (0.24%) and not significant. On the other hand, Panel C corresponds to the results obtained when the economic regime is defined according to the Saint-Louis Financial Stress Index. The sample for which the index is available covers the period from December 1993 to December 2009 and again the anomaly is only observed for the periods following normal times. The [5-1] difference between extreme quintile portfolio returns is equal to a significant -1.76% for times following normal times and to a positive but non-significant 1.80% for those following recessions.

Overall, the evidence on Table 2 supports the idea that the anomaly is not pervasive over time and is only observed after normal times. In particular, the anomaly disappears after recessions in the three cases even though the sample periods covered by each of the definitions of economic regime are different. This is a strong evidence of the influence of the economic regime in the observation of the anomaly and drives us to our hypothesis that the anomaly is not observed after recessions because during recessions there is a flight to liquidity phenomenon. To the best of our knowledge relating the anomaly to the economic conditions is pioneering. In order to test this hypothesis in the following section we use a Markov regime switching model including a variable related to innovations in market liquidity and check the influence of liquidity both for high and low idiosyncratic volatility portfolios during differing business cycles.

4.4. Results on liquidity shocks and extreme idiosyncratic risk portfolios

In this section we test the hypothesis that the anomaly is not observed during periods following recessions because of an asymmetrical impact of liquidity shocks on the extreme idiosyncratic volatility portfolios. In particular we test whether the flight to liquidity phenomenon identified by Acharya *et al.,* (2012) explains the fact highlighted in the previous section that the idiosyncratic volatility anomaly is not observed during periods following recessions.

We use a Markov regime switching model to check the relationship between the returns of both the highest and the lowest idiosyncratic risk portfolios and liquidity shocks conditional to the economic regime.[32] The model, expressed from equation [2] to equation [5], is a good candidate to represent the asymmetric dynamic behavior of stocks with differing idiosyncratic risk levels implied in our hypothesis. It is basically formed by a return structure for each type of portfolios and changes conditional on an unobservable state variable identifying the regime that follows a first order Markov chain. This is to say that the model allows for all the coefficients of the returns equations to vary between both regimes and also between types of portfolios.

For our purposes we follow the model proposed by Acharya *et al.,* (2012) to identify the effect of liquidity innovations on stocks returns so that the returns of the highest idiosyncratic volatility portfolio in regime k for k = {1,2} are assumed to be characterized by the model:

$$R_{HIVOL,t+1} = \alpha^k_{HIVOL} + \beta^k_{HIVOL,M} MKT_t + \beta^k_{HIVOL,T} TERM_t + \beta^k_{HIVOL,D} DEF_t + \beta^k_{HIVOL,S} Silliq_t + \varepsilon^k_{HIVOL,t+1}, \quad [2]$$

[32] For futher details on a Markov regime switching model refer to Hamilton (1994)

where, R_{HIVOL} corresponds to the value-weighted returns of the quintile portfolio of stocks with highest levels of idiosyncratic risk estimated using the previous 6 months, MKT is the market factor, TERM represents term structure and is computed as the difference between the market yield on U.S. Treasury securities at 10-year constant maturity and the 3-month Treasury bill rate, DEF is a variable that proxies the default premium and it is computed as the difference between Moody's yield on BAA corporate bonds and the yield on U.S. Treasury securities at 10-year maturity and Silliq is the measure of aggregate liquidity shocks. The liquidity measure corresponds to the equally-weighted average of the daily Amihud (2002) illiquidity measure averaged over each month using NYSE and AMEX stocks.[33] Liquidity shocks are measures as the innovations are obtained adjusting an AR(3) model to the index. It is important to notice that this model uses factors related to bond pricing but was shown to also account for the liquidity innovations on stock returns by Acharya *et al.,* (2012).

Similarly, the returns of the lowest idiosyncratic volatility portfolio in regime k for k = {1,2} are assumed to be characterized by the model:

$$\begin{aligned} R_{LIVOL,t+1} = \alpha^k_{LIVOL} + \beta^k_{LIVOL,M} MKT_t + \beta^k_{LIVOL,T} TERM_t + \\ \beta^k_{LIVOL,D} DEF_t + \beta^k_{LIVOL,S} Silliq_t + \varepsilon^k_{LIVOL,t+1}, \end{aligned} \quad [3]$$

where, the variables are defined as in equation 2.

The state variable s_t changes according to the Markov switching probabilities for state transition p and q such that:

$$\begin{aligned} P(s_t = 1|s_{t-1} = 1) = p \text{ and,} \\ P(s_t = 2|s_{t-1} = 2) = q, \end{aligned} \quad [4]$$

[33] Formally, Amihid measure for stock i at the end of month t is given by

$$Amih^i_t = \frac{1}{d}\sum\nolimits_{j=1}^{d} \frac{|ret^i_t|}{Vol^i_t}$$

where, d is the number of days with available data for stock i over month t, ret is the stock return and Vol its dollar volume in millions.

and variance-covariance matrix defined as:

$$\Omega_{s_t} = \begin{bmatrix} \sigma^2_{HIVOL,s_t} & \rho_{s_t}\sigma_{HIVOL,s_t}\sigma_{LIVOL,s_t} \\ \rho_{s_t}\sigma_{HIVOL,s_t}\sigma_{LIVOL,s_t} & \sigma^2_{LIVOL,s_t} \end{bmatrix}, \qquad [5]$$

also changes with the regime therefore capturing the idea that the variance of both the returns of the extreme quintiles of idiosyncratic volatility and the correlation between them may change from one regime to the other.

All the parameters of the model are estimated by Maximum Likelihood so that, in opposition to the previous section where the classification of the economic regime either as a normal or a recession period was made ex-post, the regimes are determined endogenously. This has the advantage that the classification does not depend on some particular definition of recession but has the problem that it is necessarily to actually characterize the regimes. This is to say, the model generates two regimes but it is not possible to directly argue what type of regimes they are. Applied to our case, this means we have to provide evidence that one of the regimes corresponds to recessions. To provide such evidence we regress the estimated probability of being in regime 2 against several variables related to recessions. The estimated regressions are displayed in Table 3.

In all regressions the dependent variable is the logit transform of the estimated probability of being in state 2, ($\log[(P2_t + c)/(1- P2_t + c)]$), where c = 0.5/419 to avoid problems where the probability is exactly equal to one.[34] The independent variables are (i) a dummy variable equal to one for any recession month according to the NBER Business Cycle data, (ii) the Chicago Fed National Activity Index (CFNAI) that captures the overall economic activity in the US and (iii) the Aurora Diebold Scotti Business Conditions Index (ADS). Both for the CFNAI and the ADS cases higher values of the indexes are related to better economic conditions so that we expect the sign of the coefficients related to these indexes to be negative and significant and the one of the coefficient of the NBER recession dummy to be positive and significant. The results in Table 3 confirm our hypothesis; the

[34] The logit transform is used to avoid problems related to the fact that by definition the estimated probabilities range from 0 to 1 while the linear prediction Xβ might take any real value. The constant c is defined as in Acharya *et al.,* (2012) and is necessary to avoid problem with the transforms when the estimated probability is exactly equal to 0 or 1.

coefficient relied to the NBER recession dummy is equal to 1.37 and significant at 1% confidence level, the one relied to the CFNAI is equal to -0.78 and the one related to the ADS are equal to -0.64. Both these coefficients are also significant at 1% confidence level. We therefore conclude that the regime 2 corresponds to recession and regime 1 to normal times.

Table 3: Regression analysis to identify regime 2 as recession

This table reports the regression analysis intended to characterize regime 2 of the Markov regime-switching model as recession. The dependent variable is the logit transform of the estimated probability of being in regime 2 using the sample period from January 1964 to December 2009. The independent variables are, in order, a dummy variable equal to one for NBER recession times, the Chicago Fed National Activity Index (CFNAI) and the Aurora Diebold Scotti Business Conditions Index (ADS). All the independent variables are lagged one month. * denotes significance at 10% level, ** significance at 5% level and *** significance at 1% level.

Model	*[1]*	*[2]*	*[3]*
constant	-2.72***	-2.44***	-2.50***
	[-17.88]	[-16.99]	[-18.03]
NBER Recession $_{t-1}$	1.37***		
	[3.63]		
CFNAI Index $_{t-1}$		-0.78***	
		[-5.42]	
ADS Index $_{t-1}$			-0.64***
			[-4.04]
Obs	552	552	552
Adj R2 (%)	2.16	5.24	2.71

Having identified the regimes, it is possible to analyze the results of the model displayed in Table 4 and to highlight they support our hypothesis. On the one hand, the basic result is that the returns of both the lowest and the highest idiosyncratic volatility portfolios are affected by liquidity shocks in opposite ways only during recessions. During normal times liquidity shocks affect negatively the returns of both low and high idiosyncratic volatility stocks; the coefficient relied to Silliq is significant in all cases and equal to -0.82 for the former and to -3.32 for the latter. However, during recessions the coefficient relied to the measure of liquidity shocks becomes positive and statistically significant for the low idiosyncratic risk portfolio being equal to 2.20. For the high idiosyncratic risk portfolio the coefficient equal to -5.67 is still negative but becomes not significant. Therefore, while during recessions liquidity shocks affect the returns of low idiosyncratic volatility (liquid) stocks positively they affect negatively the returns of high idiosyncratic volatility (illiquid) stocks. This result is fully consistent with our hypothesis since they reflect a flight to liquidity from stocks with higher idiosyncratic risk level to stocks with lower levels of it. This flight to liquidity should be a transient phenomenon so

Table 4: Estimation of the Markov regime-switching model

This table reports the results of the Marvoy regime-switching model. For the portfolio formed by stocks with the lowest (highest) level of idiosyncratic risk the results are found in the column labeled "low (high) idiosyncratic volatility stocks". In that case, the dependent variable is the value weighted monthly returns of the portfolio with the lowest (highest) idiosyncratic risk stocks. The independent variables are the market factor (MKT), the term structure (TERM), the default premium (DEF) and the liquidity shocks in the total market (Silliq). Regime 1 corresponds to normal times and Regime 2 to recessions. The sample covers the period between January 1964 and December 2009. * denotes significance at 10% level, ** significance at 5% level and *** significance at 1% level.

Regime 1					Parameters	
	Low idiosyncratic volatility stocks		High idiosyncratic volatility stocks			
	Coefficient	p-val	Coefficient	p-val		
Constant	-0.37*	0.06	-0.34	0.65	**p**	0.95
MKT	78.40***	0.00	137.44***	0.00	**q**	0.81
TERM	-8.71	0.12	-71.99***	0.01		
DEF	32.75***	0.00	36.77	0.41		
Silliq	-0.82***	0.00	-3.32***	0.00		

Regime 2				
	Low Idiosyncratic volatility stocks		High Idiosyncratic volatility stocks	
	Coefficient	p-val	Coefficient	p-val
Constant	0.24	0.80	0.76	0.84
MKT	77.17***	0.00	212.41***	0.00
TERM	7.81	0.78	15.74	0.90
DEF	-2.18	0.94	-53.05	0.70
Silliq	2.20**	0.05	-5.67	0.23

that in the following period stock prices should suffer a correction making the [5-1] difference in returns of extreme idiosyncratic risk portfolios smaller or even positive as observed in the previous section. On the other hand, the fact that for the regime characterized as normal times the effect of liquidity is significant for both portfolios, being more negative for the high idiosyncratic risk portfolio allow us to hypothesize the anomaly might be related to the effect of liquidity shocks on high idiosyncratic volatility stocks. Although we believe this is a noticeable path for future research a more direct proof of this issue falls outside the scope of this paper.

4.5. Conclusions

This paper highlights the idiosyncratic volatility anomaly is conditional to the state of the economy and that after recessions it might be hidden by a flight to liquidity. The paper contributes to the literature in two ways. On the one hand, it points out the relevance of past economic conditions on the anomaly showing that previous recessions end up hiding the anomaly. It also provides a plausible explanation for this based on the flight to liquidity phenomenon recently evidenced by Acharya *et al.,* (2012). In this sense our results are aligned to strong evidence provided in favor of the role of liquidity on the estimation of idiosyncratic risk by authors such as Han and Lesmond (2011) and Han *et al.,* (2011). Moreover, this paper suggests the anomaly as a whole, meaning for all regimes considered together, might be related to the asymmetric effect of liquidity shocks on the stocks forming the extreme idiosyncratic risk portfolios. This could be possible since the effect observed is that during expansions liquidity shocks decrease much more the returns of high idiosyncratic risk portfolios than those of low idiosyncratic risk. Although this second contribution opens a relevant path for further research our results do not allow us to articulate such a strong assertion and more evidence is needed in order to support it.

Chapter V: Discussion on contributions and further research

The experiences observed with previous anomalies allow anticipating that the one attached to idiosyncratic volatility will be a relevant field of research for years to come. This thesis intends to shed light to the problematic theoretical implications of the negative relationship between idiosyncratic risk and expected returns. The remainder of this chapter describes the major contributions each study has together with their limitations and possible directions for further research.

The contributions in the second chapter are threefold. On the one hand, it offers a theoretical discussion on the biases hampering the estimation of both systematic and unsystematic risk when the time interval over which returns are measured differs from the true one. This is a major issue for the idiosyncratic volatility anomaly in the sense that any bias in the estimation of the idiosyncratic risk measure could be the reason that the anomaly is observed. However, although these biases are not strange to the asset pricing literature (see for instance Levhari and Levy (1977) and Hawawini in 1983), this paper is the first to consider them as a potential explanation for the anomaly. In this sense, it poses the hypothesis that the anomaly is observed due to the co-existence of investors with different time horizons. On the other hand, the paper proposes a methodology able to estimate both the systematic and the unsystematic risk for different time intervals therefore diminishing these biases. The methodology results in the estimation of one particular idiosyncratic risk measure for each of the different groups of investors that are defined according to their investment time horizon. Finally, its third major contribution is to highlight the necessity finance discipline has of considering more complex mathematical methodologies, such as the WMRA applied in the paper, that are readily available and offer more realistic approximations to the complexity of financial markets. The theoretical case made for the use of the WMRA reveals the attractive nature of the methodology for further research on financial markets issues related to heterogeneity of market players. Some papers non referenced here also show their usefulness in treating issues such as core inflation, co-movements of stock returns and volatility spillovers. In terms of the anomaly, the paper suggests that it is not pervasive over time horizons, being relevant only for short-term investors. This result can be interpreted as evidence in favor of a more speculative nature of short-term investors that pursue very short lived investment opportunities in the market and do not consider idiosyncratic risk as a relevant determinant of their portfolio formation. The major limitation of the paper is that no time horizon shorter than 2 days can be addressed given the daily character of the data used in the analysis. In terms of intraday data nothing related to the idiosyncratic volatility anomaly has been done so that it seems like a quite direct path to pursue in future research.

In the third chapter several contributions to the literature on the idiosyncratic risk anomaly are made. On the first hand, the study shows that the anomaly should be linked to managerial decision making. In particular, the empirical results suggest the anomaly is fully accounted for when both investment and profitability controls are considered in the cross-section of stock returns. This path of research is innovative because most of the studies on the anomaly have been addressed following an investor related approach. On the other hand, the arguments offered in the critique to the accruals anomaly made by Fama and French (2006 and 2008) conclude that the negative relationship between investment and expected returns cannot be considered solely a matter of mispricing since the negative character of this relationship holds both under rational and irrational expectations. The same arguments apply to the empirical results displayed in the paper because the approach to the anomaly is based, as Fama and French (2006 and 2008) studies, on the valuation theory. Therefore, the anomaly is proven to be most likely constituted both by a component of mispricing and by a component of risk. This result is further supported in an additional test inspired by a recent paper by Gao *et al.,* (2012) who show that the anomaly is only observed during periods following high investor sentiment. Given that the negative relationship between investment and expected returns is assumed to come from valuation theory, the effectiveness of the controls for profitability and investment in accounting for the anomaly should hold also during periods of high investor sentiment. They do. This is a capital result given that this particular test does not apply to the studies of Fama and French (2006 and 2008). In this sense, it provides additional and in some sort independent support for the hypothesis that the anomaly cannot be linked solely to investor mispricing and that firm investment decisions have an influence in the anomaly. The major limitation of the analysis is its inability to disentangle how much influence each has in the anomaly. As most of the limitations, this one also constitutes an interesting path for further research.

Finally, two major contributions are made in the fourth chapter of this thesis. On the one hand, the idiosyncratic volatility anomaly is proved to be conditional to the state of the economy.[35] In this sense, the anomaly is not observed after recessions. This is an interesting contribution because it questions the extended idea that the anomaly is highly pervasive. On the other hand, the study explores the effect of liquidity on the anomaly from a novel perspective that incorporates the conditionality referred above. The study stresses how liquidity shocks occurred during recessions generate a flight to liquidity from stocks with high idiosyncratic risk to stocks with low firm specific risk. This switching movement to liquidity is offered as the source of conditionality in the anomaly since a correction in prices should happen as the market copes with the shocks. Although the paper shows that the effect of the flight to liquidity is larger than the one of the idiosyncratic volatility, its main limitation is that it only addresses a particular feature of the anomaly that is not general enough to explain it across economic regimes.

[35] Note that in its seminal papers Ang *et al.,* (2006 and 2009) show the anomaly holds both during recessions and expansions. Their definition is such that idiosyncratic risk is estimated in t while it is the period t+1 which is classified as recession or expansion. The study in this paper classifies the period t as being part of a particular regime.

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